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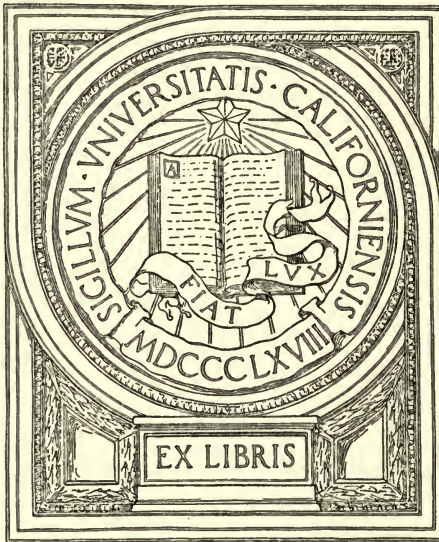


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NOTES ON LIFE INSURANCE  
BY  
EDWARD B. FACKLER.

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# NOTES ON LIFE INSURANCE

## THE THEORY OF LIFE INSURANCE PRACTICALLY EXPLAINED

An Elementary Treatise on the Principles Governing Life Insurance, and  
their Technical Application. Designed especially for the use of  
Colleges, Students and all persons interested in the subject.

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By EDWARD B. FACKLER, A.B., LL.B.

Fellow of the Actuarial Society of America

NEW YORK  
THE SPECTATOR COMPANY

1920

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## PREFACE

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IN "Notes on Life Insurance" the author has attempted to describe clearly, and at length, the general principles underlying life insurance, and then to indicate to some extent their practical application in the business. The treatment of the subject is in general the same as in Gustavus W. Smith's "Notes on Life Insurance," now out of print, the first edition of which was printed in 1870 and which became a standard life insurance text-book. This once popular book, however, is not fully applicable to the much changed insurance conditions of the present day, so it has been thought desirable to prepare a new book along similar lines.

A knowledge of arithmetic and only the most elementary algebra will be found sufficient for an understanding of all the explanations and formulas in this book.

# CONTENTS

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## CHAPTERS I-V. THEORY WITH ARITHMETICAL EXPLANATION.

- I. General Introductory Remarks.
- II. Interest and Discount, and the Mortality Table.
- III. Net Premiums.
- IV. Net Reserves.
- V. Mortality Tables and Interest Assumptions.

## CHAPTERS VI-XI, ALGEBRAIC DISCUSSION.

- VI. Elementary Formulas and the Commutation Columns.
- VII. Net Premium Formulas Stated in Commutation Symbols.
- VIII. Formulas for Net Valuation.
- IX. Annuities.
- X. Review of Formulas by Actual Calculations.
- XI. Joint Life Annuities and Insurances.

## CHAPTERS XII-XX, PRACTICAL LIFE INSURANCE.

- XII. Life Insurance Organizations.
  - XIII. Premiums and Policy Provisions.
  - XIV. Dividends.
  - XV. Governmental Supervision.
  - XVI. Company Management.
  - XVII. Industrial Insurance.
  - XVIII. Competitive Comparisons between Companies.
  - XIX. Assessment and Fraternal Insurance.
  - XX. Miscellaneous Tables and Explanations.
- Tables of Net Premiums, Reserves, Commutation Columns, Monetary Values, etc.

# INDEX

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	PAGE.
Accelerative endowment.....	114
Accumulation at interest.....	12, 51
Accumulated dividends.....	110
Actual and expected mortality.....	109
Actuaries' Table of Mortality.....	48, 50
Actuary, duties of.....	126
Additional insurance purchased with dividends.....	110
Age of insured.....	94
Agents, life insurance.....	126
American Experience Table of Mortality.....	15, 48
Amount of \$1 at compound interest.....	12, 161-165
Amount of \$1 per annum at compound interest.....	158, 161-165
Annual dividends.....	110
Annual premiums,	
Net.....	18, 169, 182, 195
Gross.....	91, 101, 102
Annual statement.....	116
Annuity,	
Certain.....	158
Deferred.....	71
Due.....	28, 166
Joint life.....	84
Last survivor.....	73
Life.....	70
Payable oftener than once a year.....	73
Survivorship.....	73
Temporary life.....	71
Application.....	127
Apportionment of dividends.....	109
Assessment insurance.....	151
Assets of insurance company.....	116
A <sub>x</sub> , definition.....	54
Beneficiary.....	94
Bonds as investments.....	122
Calculations from commutation columns.....	76-81
Carlisle Table of Mortality.....	49
Children, insurance on.....	139
Claims, death.....	94
Combined Experience Table of Mortality.....	48, 50
Commutation columns, defined.....	56
Commutation columns, figures.....	167, 180, 193
Companies, different kinds of.....	88
Competitive comparisons.....	141
Complete expectation.....	157
Compound interest.....	12
Conditions in policies.....	95
Continuous instalments.....	72
Contract, policy.....	102
Contribution plan.....	109
C <sub>x</sub> column, defined.....	58
Death claims.....	94
Deferred annuity.....	71
Deferred dividends.....	110

	PAGE.
Discount.....	12
Distribution of surplus.....	110
Dividends.....	103
Doctrine of probabilities.....	82
$D_x$ column, defined.....	56
$d_x$ column, defined.....	53
Elementary principles.....	12
Endowment insurance.....	32, 99
Endowment, pure.....	31
Estimates of dividends.....	112
Examination, medical.....	127
Examination of companies.....	121
Expectation of life.....	158
Expected mortality.....	109
Expenses.....	130
Forfeiture, non.....	96
Form, policy.....	102-107
Fraternal orders.....	153
Gain and loss exhibit.....	121
Governmental supervision.....	115
Gross premiums.....	91, 101, 102
$H^M$ Mortality Table.....	49
Immediate payment of claims.....	94
Industrial insurance.....	137
Insolvency.....	121
Inspection report.....	128
Insurable interest.....	94
Insurance,	
Assessment.....	151
Endowment.....	32, 99
Industrial.....	137
Management of.....	125
Ordinary.....	90
Term.....	24, 100
Whole life.....	21, 99
Interest, accumulation at.....	12
Interest tables.....	161-165
Investments of companies.....	122
Joint life.....	82
Annuities.....	84
Insurance.....	85
$k_x$ columns.....	69, 179, 192, 205
Lapses, effect of.....	46, 96
Last survivor annuity.....	73
Legal restrictions.....	116
Lien against policy.....	130
Limited payment policy.....	29, 99
Loadings.....	91
Loans.....	96
$l_x$ column, definition.....	52



	PAGE.
Management of life companies.....	125
Margin in premiums.....	91
Mean reserves.....	117
Medical director.....	126
Medical examination.....	127
Mixed companies.....	88
Modified preliminary-term valuation.....	134
Monetary tables.....	161-165
Mortality.....	13
Actual and expected.....	109
Amongst annuitants.....	73
Male and female.....	73
Of newly examined lives.....	129
Mortality tables.....	48
Actuaries'.....	48, 50
American Experience.....	15, 48
Carlisle.....	49
Combined Experience.....	48, 50
Explanation.....	13
H <sup>M</sup> .....	49
Northampton.....	49
Mortgage loans.....	122
Moral hazard.....	128
Mutual companies.....	88
M <sub>x</sub> columns, defined.....	59
Net premium, defined.....	18
Net premium tables.....	169, 182, 195
Non-forfeiture.....	96
Non-participating premiums.....	92
Northampton Table of Mortality.....	49
Notes, premium.....	93
N <sub>x</sub> columns, defined.....	57
Officers of life company.....	125
"Old Line" insurance.....	88
Orders, fraternal.....	153
"Ordinary" insurance.....	90
Ordinary whole life policy.....	28, 99
Participation in surplus.....	92
Plans of insurance.....	99
Policy form.....	102-107
Policy, loan on.....	96
Population mortality tables.....	49
Preliminary-term valuation.....	131
Premium.....	11
Extra.....	130
Gross.....	91, 101, 102
Net.....	18, 169, 182, 195
Return.....	62, 100
Semi-annual and quarterly.....	93
Single.....	22
Premium notes.....	93
Present value of \$1.....	13, 161-165
Present value of \$1 per annum.....	158, 161-165
Probability, laws of.....	82
Probability of living a term of years.....	156
Procedure of obtaining policy.....	11

	PAGE.
Proofs of death.....	94
Pure endowment.....	31
Quarterly, annuity payable.....	75
Quarterly premiums.....	93
Rate of interest assumed.....	48
Reinsurance of risks.....	129
Renewable term.....	100
Report, annual.....	116
Report, inspection.....	128
Reserve tables.....	170, 183, 196
Reserves, formulas for.....	65
Reserves.....	33
Modified preliminary-term.....	134
Preliminary-term.....	131
Select and Ultimate.....	135
Return premiums.....	62, 100
$R_x$ columns.....	59
Savings in,	
Expenses.....	109
Interest.....	108
Mortality.....	109
“Selection”.....	129
Select and Ultimate valuation.....	135
Semi-annual premiums.....	93
Semi-tontine dividends.....	110
Single premiums.....	169, 182, 195
Size of companies.....	141
Special forms of policy.....	101
Statement, annual.....	116
Stipulated premium companies.....	153
Stock companies.....	88
Stock investments.....	123
Sub-standard risks.....	130
Surplus, source and distribution of.....	108
Surrender values.....	96
Survivorship annuity.....	73
$S_x$ column.....	59
Tables, list of.....	160
Temporary annuity.....	71
Term insurance.....	24, 100
Terminal reserves.....	45
Tontine insurance.....	111
Underaverage lives.....	130
$u_x$ columns.....	69
Valuation of policy liabilities.....	117
Valuation,	
Modified preliminary-term.....	134
Preliminary-term.....	131
Select and Ultimate.....	135
Values, surrender.....	96
Vie probable.....	158
Whole life insurance.....	21, 99
Women, insurance on.....	130



## CHAPTER I.

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### GENERAL INTRODUCTORY REMARKS.

LIFE insurance has now become established as one of the greatest economic triumphs of this progressive age. It is long past the experimental stage, and is regarded as a practical necessity in civilized society. Each business day of the year on the average about \$1,000,000 is paid to policy-holders by the life insurance organizations of the United States. There are outstanding in this country at present contracts calling for the payment to individuals in the future of over \$13,750,000,000 by what are called "regular" or "legal reserve" companies, and of \$8,000,000,000 payable by other organizations.

The growth of the institution in this country has been very rapid. Forty years ago it was distrusted because its purpose and workings were not understood by the great mass of the people, and many had conscientious scruples against entering into a contract which seemed to them to be an interference with Providence. A large number of men have spent their lives in teaching their fellows that life insurance is not wrong, but on the other hand is simply a safe and common-sense way of capitalizing a man's future earning powers; and so of providing him the means of undertaking family and business responsibilities which it would be folly to attempt without such protection.

That insurance, taken in the abstract, is analogous to a game of chance, cannot be denied; but there is a very important distinction, which absolutely removes from the business the stigma of gambling. When a man gambles, he ventures a small sum in the hope that chance will award him a large sum in return. He has no interest in the factors which decide the event beyond the wish that chance will turn his way. If chance turns against him, his circumstances and those of the persons dependent on him are altered only to the extent of the loss of the comparatively small sum "staked." If chance favors him he is simply enriched from the losses of those whom chance did not favor.

With life insurance the case is quite different. Here, though

an insured man ventures a small sum,—his premium,—on the chance that a larger sum will be paid, the event which will make that larger sum payable,—his death,—is most emphatically one which he does not wish to happen, and which will involve financial loss to those dependent on him. If chance “favors” him, the insurance money will only serve to make good in part the loss occasioned by his death, while if chance “goes against him,” and he lives, he is quickly reimbursed, through his continued power to earn money, for the sum previously spent for protection against the loss of that power.

It is of course apparent that this extremely beneficial contract is capable, like most other excellent things, of abuse, and that a life insurance policy can be made the basis of a most revolting sort of gamble, when the prospective beneficiary of the policy instead of desiring the continued life of the insured person, wishes, or strives to bring about, his death. This evil sprang up early in the history of life insurance, but was speedily recognized, and laws were enacted, which, together with the efforts of the insurance companies, have almost entirely suppressed the practice.

All kinds of insurance, and there are many, are based on the laws of probability, but, of them all, life insurance presents the best basis for contracts involving the certain payment of money. Thus, in fire insurance the insurance money is payable if the property burns. In marine insurance it is payable if the ship is lost or damaged. But the house may never burn, and the ship may never be lost. In the case of life insurance, however, the insurance money is payable at the death of the person insured, and the element left to be decided by chance is not whether the man will die, but how soon he will die. It is evident that the older a man is, the more likely he is to die “soon,” and the younger he is the more likely he is to survive for a number of years.

Fire and marine insurance underwriting, being for short periods, is sometimes done by individuals or partnerships. A contract of life insurance, however, often is such that it may be continued, not fully consummated, for two generations, or even longer. For this reason, among others, life insurance is always effected through the medium of corporations or associations, whose existence may be perpetual, and thus independent of the continued existence of any particular person or small group of persons.

To every contract of life insurance there are at least two parties. One of these is the insurance company, and the other is the person whose life is insured. There is a great variety of contracts, but their essential characteristic is to provide for a payment to be made at the death of the insured. In some cases the period during which death must occur for any payment to be made is limited; in others it is not. Many contracts provide, in addition to the obligation to make a payment at death during a certain term, that a payment be made at the close of that time in case the insured be living. All of these things that have been mentioned so far are promises by the life insurance company. The consideration required by the insurance company for such a contract consists of two parts. One of these is truthful information on certain points, principally as to the insured's occupation, health and family history, which are regarded as necessary in order that the insurance company may properly classify its risks. The other part of the consideration is a sum of money, to be paid to the company periodically either during the entire life of the insured or for a limited term, according to the form of contract.

The written contract issued by the insurance company is known as the "policy;" the information as to the insured's personal physical condition, etc., is known as the "application;" and the definite sum of money payable by the insured is called the "premium." The policy is issued in consideration of the application and the premiums. From the application and the report of a personal examination by a physician the officers of the company decide whether the "applicant" can be considered as being such a risk as the company may safely insure. Then if he is accepted the policy is issued, and if the first premium is paid the contract goes into force. An understanding of the calculation, by which an insurance company determines what premium it must charge in order to meet its obligations, involves an examination of the principles upon which life insurance is based, and we will now enter upon a discussion of these theories.



## CHAPTER II.

INTEREST AND DISCOUNT, AND THE MORTALITY  
TABLE.

ALL life insurance calculations depend on two main facts. One of these is that invested funds will increase through interest earnings, and the other is that it is possible to foretell with a fair degree of certainty about how many deaths will occur in successive years out of a given number of persons now living, as to whom certain information has been obtained.

INTEREST AND DISCOUNT:—If it is known that \$100 invested in an interest-bearing security will yield 3 per cent. interest, or \$3, at the close of a year from the present, we can assume that on these conditions \$100 paid now is the exact equivalent of \$103, principal and interest, to come due at the close of one year. Again, if we assume that the \$3 earned the first year will immediately be invested, together with the principal of \$100, at 3 per cent. interest, there will be earned on the \$103 at the end of one year further, \$3.09; in other words, on this hypothesis,  $\$100 + \$3 + \$3.09$ , which equals \$106.09, to be paid at the close of two years from the present, is the exact equivalent of \$100 paid now. Carrying this further on the same hypothesis, \$109.27 paid at the end of 3 years, or \$112.55 paid at the end of 4 years, or \$115.93 paid at the end of 5 years, etc., are each of them the exact equivalent of \$100 paid now. In any life insurance calculation a certain rate of interest is thus assumed as being exactly according to the actual facts, and then the various sums above indicated are regarded as exact equivalents for the purposes of the calculation. Extended tables of such values, at various rates of interest, are given in the latter part of the book, the unit of principal in that case being taken as \$1, instead of \$100—all other values being, of course, in proportion.

When we wish to find the amount payable at the expiration of a certain period which is exactly equivalent to a given sum paid now, it is convenient to use a table constructed as above

described, having as a unit, \$1, \$10, or \$100, as the case may be, paid now. If, however, it is desired to find the present value of a sum which is to be paid at the close of a certain period, the above mentioned table is not convenient to use. What we wish in such a case is a table showing, in figures expressed to several decimal places, the exact equivalent in present value of the sum of \$1, \$10, or \$100 payable at a certain designated time in the future. The values in this case are to be found by simple proportion from the values obtained in the previous table. Thus, if \$103 paid at the end of one year from the present is the equivalent of \$100 paid now, \$100 paid at the end of one year will be the equivalent of  $\frac{100}{103}$  of \$100. This, expressed in decimals, is \$97.0874. Similarly the present value of \$100 paid at the end of 2 years is equal to  $\frac{100}{106.09}$  of \$100, or \$94.2596, etc. Extended tables of such present values are given among the tables at the end of this book on the basis of \$1 as a unit. It is clear that the values in these tables are merely an adaptation, in a more convenient form, of the corresponding values in the previously mentioned tables, and the figures given may easily be shown to be correct by improving them at compound interest for the period designated. These latter tables are the more generally used in life insurance calculations.

It should be noticed that the present value of \$1 payable 10 years hence at 3 per cent. interest (\$0.744,094) is greater than at  $3\frac{1}{2}$  per cent. (\$0.708,919); greater at  $3\frac{1}{2}$  per cent. interest than at 4 per cent. (\$0.675,564), and so on. In other words, the present value decreases as the rate of interest increases, and vice versa. If we disregard the fact that invested funds earn interest, or, what amounts to the same thing, if we assume that the rate of interest earned is zero per cent., we will have, as a result, that \$1 payable at any time in the future is the equivalent of the full \$1 paid now. In some places in our discussion it will be found convenient to make this assumption.

It is very important to fix in mind at the outset the idea and the reasonableness of considering amounts payable at different times under certain conditions as the exact equivalents of one another, for this assumption is continually being made in life insurance calculations.

**MORTALITY TABLE—ITS FORM AND USE:**—A mortality table is an arrangement in concise, convenient form, of facts deduced

by experts from extended observation of statistics as to the numbers living and dying among a large number of people. A great many such tables have been constructed, and their general agreement in results has served to increase the confidence placed in them. The table given on the following page, known as the "American Experience Table of Mortality," was constructed from actual experience with insured lives in this country and, while not absolutely perfect, it has been found to be a conservative and satisfactory basis for life insurance calculations.

*American Experience Table of Mortality.*

Age.	Number Living.	Number of Deaths.	Death Rate per 100.	Age.	Number Living.	Number of Deaths.	Death Rate per 100.
10	100 000	749	0.75	53	66 797	1 091	1.63
11	99 251	746	.75	54	65 706	1 143	1.74
12	98 505	743	.75	55	64 563	1 199	1.86
13	97 762	740	.76	56	63 364	1 260	1.99
14	97 022	737	.76	57	62 104	1 325	2.13
15	96 285	735	.76	58	60 779	1 394	2.29
16	95 550	732	.77	59	59 385	1 468	2.47
17	94 818	729	.77	60	57 917	1 546	2.67
18	94 089	727	.77	61	56 371	1 628	2.89
19	93 362	725	.78	62	54 743	1 713	3.13
20	92 637	723	.78	63	53 030	1 800	3.39
21	91 914	722	.79	64	51 230	1 889	3.69
22	91 192	721	.79	65	49 341	1 980	4.01
23	90 471	720	.80	66	47 361	2 070	4.37
24	89 751	719	.80	67	45 291	2 158	4.76
25	89 032	718	.81	68	43 133	2 243	5.20
26	88 314	718	.81	69	40 890	2 321	5.68
27	87 596	718	.82	70	38 569	2 391	6.20
28	86 878	718	.83	71	36 178	2 448	6.77
29	86 160	719	.83	72	33 730	2 487	7.37
30	85 441	720	.84	73	31 243	2 505	8.02
31	84 721	721	.85	74	28 738	2 501	8.70
32	84 000	723	.86	75	26 237	2 476	9.44
33	83 277	726	.87	76	23 761	2 431	10.23
34	82 551	729	.88	77	21 330	2 369	11.11
35	81 822	732	.89	78	18 961	2 291	12.08
36	81 090	737	.91	79	16 670	2 196	13.17
37	80 353	742	.92	80	14 474	2 091	14.45
38	79 611	749	.94	81	12 383	1 964	15.86
39	78 862	756	.96	82	10 419	1 816	17.43
40	78 106	765	.98	83	8 603	1 648	19.16
41	77 341	774	1.00	84	6 955	1 470	21.14
42	76 567	785	1.03	85	5 485	1 292	23.56
43	75 782	797	1.05	86	4 193	1 114	26.57
44	74 985	812	1.08	87	3 079	933	30.30
45	74 173	828	1.12	88	2 146	744	34.67
46	73 345	848	1.16	89	1 402	555	39.59
47	72 497	870	1.20	90	847	385	45.45
48	71 627	896	1.25	91	462	246	53.25
49	70 731	927	1.31	92	216	137	63.43
50	69 804	962	1.38	93	79	58	73.42
51	68 842	1,001	1.45	94	21	18	85.71
52	67 841	1,044	1.54	95	3	3	100.00



The meaning of this table may be said to be as follows:—Out of 100,000 persons now exactly 10 years old, of the class of lives acceptable to insurance companies, 749 will die within one year from this time, i.e., before reaching their 11th birthday, and the remaining 99,251 persons will live through the year, and become 11 years old.

Out of 99,251 persons exactly 11 years old, 746 will die within one year thereafter, and 98,505 will live through the year. We can assume or not, as we choose, that these 98,505 persons, who thus become 12 years old, were members of the group of 100,000 who were exactly 10 years old two years earlier.

Similarly, out of 92,637 persons exactly 20 years old, 723 will die within one year, or before reaching the age of 21, and 91,914 will live through that year. If we sum the first eleven lines of the column "Number of Deaths," ending with 723, giving 8,086, and deduct this from 100,000 we have as a result 91,914. Therefore we may consider, if desirable, that the 92,637 persons 20 years old, the 723 who die before becoming 21, and the 91,914 who attain age 21 were all members of a group of 100,000 persons exactly 10 years old, ten or eleven years before.

In the same way the 5,485 persons becoming 85 years old, the 1,292 dying within a year, and the 4,193 who survive to age 86 can be considered to have been members of a group of 92,637 persons aged 20, or of 100,000 persons aged 10 living many years before.

An analysis of this table shows us that, besides telling how many deaths will occur each successive year in a group starting at the same age, the table shows also the number of survivors of the group at the close of each year. We shall use both of these columns in our calculations hereafter. It is to be remembered however that if we had, e. g., at age 10, 100,000 persons known to us by name, we are not saying that we would know which the 749 are who would die within a year. Theoretically, and so far as we know, each person of the 100,000 is as likely to be of the 749 who die as any other.

Finally, if we sum the column headed "Number of Deaths" from beginning to end we will find the total to be 100,000, or the number at the head of the "Number Living" column. This simply means that, according to this table, no person will live to be quite 96 years old. The compiler of the table knew it to be



a fact that some few persons do actually live to be 96, or even several years older; but as there had been so little experience at such advanced ages, he decided to consider the limit of life as just short of 96 years.

The foregoing long description has been given at this point to allow of short unexplained references to the table hereafter, and should be thoroughly mastered. This set of ideal assumptions may seem somewhat visionary, but later they will be shown to serve very well as a basis for practical calculations.

## CHAPTER III.

## NET PREMIUMS.

THE premium, which an insured person pays to the insurance company at stated intervals, consists of two elements,—the “net premium” and the “loading” or “margin.” This latter portion is intended primarily to cover the expenses of carrying on the business. For the present, however, we will disregard this element of “loading,” for, while it is practically necessary in the premium, it has no place in an elementary explanation of the theoretical foundation of the business. We will first seek to find the amount for which, disregarding all matters of expense, an insurance company can afford to make a contract to pay a stated sum of money at a man’s death.

Premiums are always payable to the company in advance. Death claims,—i. e. amounts insured which the insurance company must pay because death has occurred—are considered in calculations as though they are to be paid only at the close of the policy year during which death occurs: i. e. just prior to the next anniversary of the policy’s issue. These two points must be remembered.

We will now calculate the “Net Premium for One Year Term Insurance of \$1, at age 50.” The question is:—“For what sum can an insurance company agree to pay \$1 at the end of one year to the representatives of X, now 50 years old, provided his death occur during one year from the present?” We premise here, as elsewhere, that persons to be insured have satisfied the company that they are acceptable risks.

We will assume that X is one of 69,804 persons all 50 years old who each apply for \$1 insurance for one year. We have taken, for convenience, the number shown in the column “Number Living” at age 50 in the mortality table. We do not know which of these people, of whom X is one, will die; but we do know, from the mortality table, that 962 of them will die before the close of one year. The number opposite age 50 in the “Number of Deaths” column tells us so; meaning that if the insurance company accepts

these 69,804 people as risks for \$1 insurance each, it will have to pay out in all at the close of the year, to the representatives of the 962 who die, \$962. The payment of this sum at that time would be a certainty. Therefore, if the company's funds earned no interest it would need to get from the 69,804 persons at the beginning of the year just \$962 altogether, in order to be able to pay that sum out at the end of the year. We are, however, going to assume in all explanatory examples that money earns just 3 per cent. interest. Then we may consider, according to the 3 per cent. present value table, that for each \$1 of the \$962 which it must pay a year hence, the company need receive from the insured persons only \$0.970,874, or a fraction over 97 cents. Therefore it need have on hand now only  $962 \times \$0.970,874$ , which is equal to \$933.98.

Now as X is one of the group of the 69,804 who are insured, and as he therefore may be one of the 962 persons who it is known will die, he should bear the same share of the cost of insurance as any one of the rest of them. As they are all coming into the company on the same basis and each may be one of the 962 who die, each should be charged the same premium. As \$933.98 and a fraction of a cent is to be paid by the group of 69,804 persons, the premium to be paid by X and each other of the group is therefore  $\$933.98 \div 69,804$ , or \$0.013,38. The receipt of this \$0.013,38 from each would just allow the company to meet its obligations to the representatives of the 962 persons who die. The 68,842 persons who survive the year receive no money return. They have received their money's worth of insurance protection.

It is clear that if a company is to promise to pay a certain sum at a man's death during a year, it should, in order to fully protect itself, make the same or similar contracts with a considerable number of other persons, so that the premiums received during the year will suffice to pay the insurance in case he dies. The number thus insured, however, need not be the exact number in the mortality table, for the mortality table simply expresses ratios. If, for instance, the number taken were one-half that in the mortality table, or 34,902, the deaths would be one-half, or 481, and the premium to be paid by each person would be unchanged. No matter what the number of persons under consideration, the proportion of deaths in a year would theoretically be  $\frac{962}{69,804}$  of that number. In other words, each individual's chance of dying has the value of

$\frac{.962}{69,804}$ , so the value of his insurance for \$1 is that fraction of the present value of \$1 payable certainly a year hence. For \$1,000 insurance the premium to be paid by each individual would be 1,000 times as much, or \$13.38.

For the premium for an insurance of \$1 for one year at age 51 the calculation is similar to the above. At this age in the mortality table we find that 1,001 persons, out of a group of 68,842 who were living at the outset, die before the close of the year, so that \$1,001 would be payable by the company at the end of a year if 68,842 persons had each been insured for \$1. The present value of the \$1,001 at the beginning of the year is  $1,001 \times \$0.970,874$ , or \$971.845, and the amount to be collected from each person insured would be  $\frac{1}{68,842}$  of that, or \$0.014,12.

From the standpoint of the individual, no matter how many were insured, the value of the probability of his dying would be  $\frac{1,001}{68,842}$  and the premium for his insurance of \$1 would be  $\frac{1,001}{68,842}$  of the present value of \$1 payable certainly a year hence. As before, his premium for \$1,000 insurance would be 1,000 times that for \$1, or \$14.12.

In the following table is given the premium for \$1,000 insurance for one year at each age from 20 up, with the same table of mortality and rate of interest assumed in the foregoing examples. The student will do well to work the premium out for himself at one or more ages, following the same rules as shown in the examples, and using the figures in the mortality table opposite to those ages.

It will be seen that the premium increases with each year of age, and becomes, at age 95, \$970.87, which is merely the present value of \$1,000 due a year hence, the death within a year of any one insured at that age being accepted as a certainty.



*Net Premiums, on the Basis of American Experience Table and  
3 Per Cent. Interest, for \$1,000 Insurance for One Year.*

Age.	Premium.	Age.	Premium.	Age.	Premium.
20	7.58	45	10.84	70	60.19
21	7.63	46	11.23	71	65.69
22	7.68	47	11.65	72	71.59
23	7.73	48	12.14	73	77.84
24	7.78	49	12.72	74	84.49
25	7.83	50	13.38	75	91.62
26	7.89	51	14.12	76	99.33
27	7.96	52	14.94	77	107.83
28	8.02	53	15.86	78	117.31
29	8.10	54	16.89	79	127.90
30	8.18	55	18.03	80	140.26
31	8.26	56	19.31	81	153.98
32	8.36	57	20.71	82	169.22
33	8.46	58	22.27	83	185.98
34	8.57	59	24.00	84	205.20
35	8.69	60	25.92	85	228.69
36	8.82	61	28.04	86	257.94
37	8.97	62	30.38	87	294.19
38	9.13	63	32.95	88	336.59
39	9.31	64	35.80	89	384.33
40	9.51	65	38.96	90	441.31
41	9.72	66	42.43	91	516.96
42	9.95	67	46.26	92	615.79
43	10.21	68	50.49	93	712.79
44	10.51	69	55.11	94	832.18
				95	970.87

NET SINGLE PREMIUM FOR WHOLE LIFE INSURANCE:—We are now better prepared to consider the calculations for insurance for the whole of life. We again assume the age of the insured as 50 at the time the insurance begins. Turning to the mortality table we see that of 69,804 persons aged 50, 962 will die in the first year following, 1,001 will die in the second year, 1,044 in the third year, and so on, the last three dying in the 46th year. If the 69,804 persons each take out \$1 insurance for life, the company will in the long run pay out just \$69,804, but not all at once. It will be liable to pay out \$962 at the end of one year from now, an additional \$1,001 a year later, \$1,044 two years later, and so on.

We now will calculate the "Net Single Premium at age 50 for Whole Life Insurance of \$1." The question is: "What sum paid now to an insurance company by X, 50 years old, will just allow it to pay \$1 at his death?" As before, we will assume that X is one of 69,804 persons 50 years old who are to be insured for \$1 each. Our previous example shows us just how much would have to be collected from him in order that he should bear his fair share of the death claims which would fall due within one year. We found this to be \$0.013,380. In the second year, according to the mortality table, 1,001 persons will die, calling for \$1,001 at the end of that year from the insurance company. This \$1,001 being payable certainly two years from this time should be discounted for two years. In other words, the insurance company should collect from the group of insured persons the present value of \$1,001 payable two years hence. According to our discount table the present value at 3 per cent. interest of \$1 payable two years hence is \$0.942,596, and therefore the present value of the \$1,001 would be  $1,001 \times \$0.942,596 = \$943.54$  approximately. As each one of the 69,804 persons is liable to be one of the group of 1,001 who die in this second year, each one should bear an equal portion of the cost, or  $\frac{1}{69,804}$  of \$943.54, which is \$0.013,517. Similarly, the \$1,044 which will be payable at the end of three years should be discounted for that period, the result being \$955.41, and this sum likewise should be assessed equally on each of the 69,804 persons, making the payment by each \$0.013,687, and so on, for the death claims which will fall due in each year following.

In the subjoined table this calculation is shown in full. Here X, and each other person of the 69,804, will in this way be charged sufficient to pay for his share of the death claims, no matter when he may die, the sum of these items for the successive years thus making up the single premium, \$0.555,217. The operations might be materially shortened by adding together the present value of the death claims which would fall due in all future years, and then once for all dividing by the number 69,804, which would give us the same result. It is thought, however, that the foregoing method of explanation is somewhat easier to grasp.

*Calculation of the Net Single Premium at Age 50 for a Whole Life Insurance of \$1, based on American Experience Table and 3 per cent. interest.*

Year of insurance.	Age attained beginning year.	(1) Present value of \$1, payable certainly at end of years indicated.	(2) Tabular number of deaths in year.	(3) Product of (1) × (2). Present value of total sum payable by Company at end of years indicated.	(4) Tabular number living at age 50.	Quotient of (3) ÷ (4). Share of the sum in Col. (3) to be paid now by each person.
1	50	\$0.970 874	962	\$933.980 788	69 804	\$0.013 380
2	51	.942 596	1 001	943.538 596	69 804	.013 517
3	52	.915 142	1 044	955.408 248	69 804	.013 687
4	53	.888 487	1 091	969.339 317	69 804	.013 887
5	54	.862 609	1 143	985.962 087	69 804	.014 125
6	55	.837 484	1 199	1 004.143 316	69 804	.014 385
7	56	.813 092	1 260	1 024.495 920	69 804	.014 677
8	57	.789 409	1 325	1 045.966 925	69 804	.014 984
9	58	.766 417	1 394	1 068.385 298	69 804	.015 306
10	59	.744 094	1 468	1 092.329 992	69 804	.015 649
11	60	.722 421	1 546	1 116.862 866	69 804	.016 000
12	61	.701 380	1 628	1 141.846 640	69 804	.016 353
13	62	.680 951	1 713	1 166.469 063	69 804	.016 711
14	63	.661 118	1 800	1 190.012 400	69 804	.017 048
15	64	.641 862	1 889	1 212.477 318	69 804	.017 370
16	65	.623 167	1 980	1 233.870 660	69 804	.017 676
17	66	.605 016	2 070	1 252.383 120	69 804	.017 941
18	67	.587 395	2 158	1 267.598 410	69 804	.018 159
19	68	.570 286	2 243	1 279.151 498	69 804	.018 325
20	69	.553 676	2 321	1 285.081 996	69 804	.018 410
21	70	.537 549	2 391	1 285.279 659	69 804	.018 413
22	71	.521 893	2 448	1 277.594 064	69 804	.018 303
23	72	.506 692	2 487	1 260.143 004	69 804	.018 053
24	73	.491 934	2 505	1 232.294 670	69 804	.017 654
25	74	.477 606	2 501	1 194.492 606	69 804	.017 112
26	75	.463 695	2 476	1 148.108 820	69 804	.016 448
27	76	.450 189	2 431	1 094.409 459	69 804	.015 678
28	77	.437 077	2 369	1 035.435 413	69 804	.014 833
29	78	.424 346	2 291	972.176 686	69 804	.013 927
30	79	.411 987	2 196	904.723 452	69 804	.012 961
31	80	.399 987	2 091	836.372 817	69 804	.011 982
32	81	.388 337	1 964	762.693 868	69 804	.010 926
33	82	.377 026	1 816	684.679 216	69 804	.009 809
34	83	.366 045	1 648	603.242 160	69 804	.008 642
35	84	.355 383	1 470	522.413 010	69 804	.007 484



*Calculation of the Net Single Premium at Age 50, etc.—Continued.*

Year of insurance.	Age attained beginning year.	(1) Present value of \$1, payable certainly at end of years indicated.	(2) Tabular number of deaths in year.	(3) Product of (1) × (2). Present value of total sum payable by Company at end of years indicated.	(4) Tabular number living at age 50.	Quotient of (3) ÷ (4). Share of the sum in Col. (3) to be paid now by each person.
36	85	\$0.345 032	1 292	\$445.781 344	69 804	\$0.006 386
37	86	.334 983	1 114	373.171 062	69 804	.005 346
38	87	.325 226	933	303.435 858	69 804	.004 347
39	88	.315 754	744	234.920 976	69 804	.003 365
40	89	.306 557	555	170.139 135	69 804	.002 437
41	90	.297 628	385	114.586 780	69 804	.001 642
42	91	.288 959	246	71.083 914	69 804	.001 018
43	92	.280 543	137	38.434 391	69 804	.000 551
44	93	.272 372	58	15.797 576	69 804	.000 226
45	94	.264 439	18	4.759 902	69 804	.000 068
46	95	.256 737	3	0.770 211	69 804	.000 011

Sum (Net Single Premium).....\$0.555 217

NOTE.—Calculation with greater accuracy (involving more decimal places) gives as a result \$0.555 215.

As in the case of the premium for insurance for one year, we are not dependent on the number of persons who enter into the contract for insurance, and the premium for a greater amount of insurance than \$1 would be directly proportional to that for \$1; in other words, the single premium for \$1,000 of whole life insurance would be  $1,000 \times \$0.555,217 = \$555.22$ . Tables of these single premiums on different mortality tables and at various rates of interest are given in the latter part of this book.

TERM INSURANCE:—From our table showing the manner of calculating the net single premium for whole life insurance of \$1 at age 50, it is easy to see how the result would be changed if the insurance were to cover a term of years instead of life. Thus, if we wish the net single premium for \$1 insurance at age 50 for 20 years only, we would have to consider only the first 20 lines of the preceding table. In those 20 lines the death claims for the first 20 years are properly discounted, and equally divided among the insured persons, and the sum of the first 20 amounts in the extreme right-hand column would give us the net 20-year term premium. In this case the result is \$0.317,595. For \$1,000



insurance the premium would be \$317.60. A number of insured persons would survive the 20 years and many deaths would occur thereafter, but as we have limited to 20 years the time in which death must occur for anything to be paid, the history of the persons outliving the 20 years is of no consequence in this connection.

**INSURANCE BY ANNUAL PREMIUMS:**—We have shown the method of arriving at the amount to be paid down in one sum, for insurance for the whole of life. This was necessary as the first step in the explanation of the subject, but as a matter of fact few insurances are paid for in that manner. Premiums are generally made payable in many periodical instalments. Sometimes they are to continue during life, and sometimes only for a limited term during the life of the insured: theoretically they are always assumed to be payable at the beginning of each year of the period during which they are to be paid. It is obvious that no matter how such periodical premiums are made payable, they must be exactly equivalent to the corresponding net single premium, for otherwise the company would be collecting either more or less than enough to pay death claims as they fall due. The question which we will then consider is: "What sum payable at the beginning of each year during the life of a man aged 50 is the exact equivalent of the above single premium for whole life insurance?"

In our study of the mortality table we learned that it not only shows us how the deaths will happen but also how many persons of the original group will survive the various years following. We will now give our attention wholly to the column "Number Living," entering it at age 50.

Let us now compute the present value, to an insurance company, of \$1 to be paid by each of a group of 69,804 persons 50 years old, now and at the beginning of each year hereafter, so long as any are living. The total value of \$1 from each of them now is obviously \$69,804. Only 68,842 are living a year from now, so only \$68,842 is paid the company. This \$68,842 is due a year hence, so we discount it for one year and  $68,842 \times \$0.970,874 = \$66,836.91$  is its equivalent present value. Similarly, \$67,841 will be paid the company two years hence by those who become 52 years old. This sum discounted for two years, or  $67,841 \times \$0.942,596 = \$63,946.66$ , its equivalent present value, etc. At this point is given a table showing the steps already outlined for the whole of life, to be followed by an explanation of the further steps.

*Calculation of Present Value, based on the American Experience Table and 3 per cent. interest, of \$1 paid now and at the beginning of each year of life by each of a group of 69,804 persons 50 years old and their survivors.*

YEAR.	Attained age at beginning of year.	(1) Present value of \$1 to be paid certainly at beginning of years indicated.	(2) Tabular number living at beginning of each year.	Product of (1)×(2). Present value of sum to be received by company in each year.
1	50	\$1.000 000	69 804	\$69 804.000 000
2	51	0.970 874	68 842	66 836.907 908
3	52	.942 596	67 841	63 946.655 236
4	53	.915 142	66 797	61 128.740 174
5	54	.888 487	65 706	58 378.926 822
6	55	.862 609	64 563	55 692.624 867
7	56	.837 484	63 364	53 066.336 176
8	57	.813 092	62 104	50 496.265 568
9	58	.789 409	60 779	47 979.489 611
10	59	.766 417	59 385	45 513.673 545
11	60	.744 094	57 917	43 095.692 198
12	61	.722 421	56 371	40 723.594 191
13	62	.701 380	54 743	38 395.645 340
14	63	.680 951	53 030	36 110.831 530
15	64	.661 118	51 230	33 869.075 140
16	65	.641 862	49 341	31 670.112 942
17	66	.623 167	47 361	29 513.812 287
18	67	.605 016	45 291	27,401.779 656
19	68	.587 395	43 133	25,336.108 535
20	69	.570 286	40 890	23 318.994 540
21	70	.553 676	38 569	21 354.729 644
22	71	.537 549	36 178	19 447.447 722
23	72	.521 893	33 730	17 603.450 890
24	73	.506 692	31 243	15 830.578 156
25	74	.491 934	28 738	14 137.199 292
26	75	.477 606	26 237	12 530.948 622
27	76	.463 695	23 761	11 017.856 895
28	77	.450 189	21 330	9 602.531 370
29	78	.437 077	18 961	8 287.416 997
30	79	.424 346	16 670	7 073.847 820
31	80	.411 987	14 474	5 963.099 838
32	81	.399 987	12 383	4 953.039 021
33	82	.388 337	10 419	4 046.083 203
34	83	.377 026	8 603	3 243.554 678
35	84	.366 045	6 955	2 545.842 975

*Calculation of Present Value, based on the, etc.—Continued.*

YEAR.	Attained age at beginning of year.	(1) Present value of \$1 to be paid certainly at beginning of years indicated.	(2) Tabular number living at beginning of each year.	Product of (1) × (2). Present value of sum to be received by company in each year.
36	85	\$0.355 383	5 485	\$1 949.275 755
37	86	.345 032	4 193	1 446.719 176
38	87	.334 983	3 079	1 031.412 657
39	88	.325 226	2 146	697.934 996
40	89	.315 754	1 402	442.687 108
41	90	.306 557	847	259.653 779
42	91	.297 628	462	137.504 136
43	92	.288 959	216	62.415 144
44	93	.280 543	79	22.162 897
45	94	.272 372	21	5.719 812
46	95	.264 439	3	0.793 317

Total Present Value of Sums Receivable.....\$1 065 973.172 166

In the foregoing table the present value of the amounts which should be received now and at the beginning of each year hereafter from the group of 69,804 and its survivors is shown in the last column, and the sum of these present values given at the end of the table is the total present value to the insurance company of all the payments to be made by the group and its survivors.

At the outset the company has no means of knowing how many payments of \$1 it will receive from any one individual, but it does know that it will receive the sum indicated from the whole group. If, therefore, it were desired to make a settlement with the company in one lump sum for all these payments, the group at the present time would have to pay the company exactly this present value, and as each person had come into the contract on exactly the same basis, the share of each should be exactly equal. In other words, each person should pay the company the sum of  $\frac{1}{69,804}$  of \$1,065,973.172,166, which is \$15.271. This sum is therefore the present value to the company of payments of \$1 now and in the beginning of each year hereafter of the life of a person now 50 years old. If the payments were \$100 each instead of \$1, the present value would be 100 times \$15.271, or \$1,527.10. If the payments were one cent each year, instead of \$1, the present value would be  $\frac{1}{100}$ th, or \$0.152,71, and so on for any amount.



A series of payments annually during life is known as a "life annuity" or "annuity." The life series of payments we here have under consideration is called an "annuity-due," because its first payment is immediately due and the other payments follow at the beginning of each year of life.

NET ANNUAL PREMIUM FOR WHOLE LIFE INSURANCE:—Previously we ascertained the single premium, or the sum necessary for the company to have on hand now on account of a man 50 years old, to insure him for life for \$1,000, and we have now found the present value of payments of \$1 now and at the beginning of each year of his life hereafter. From these two amounts we can find what sum payable now and at the beginning of each year of his future life will have a present value exactly equal to the present value of his \$1,000 insurance, by dividing the single premium for \$1,000 insurance by the present value of the series of payments of \$1. In this way we find the number of times the value of the annual payments of \$1 is contained in the value of the insurance, and determine the number of series of payments of \$1 each which will be equivalent to the single premium. Carrying this out, gives us  $\$555.215 \div \$15.271 = \$36.36$ , which is the net annual premium for whole life insurance of \$1,000 at age 50. For \$1 insurance the premium would be \$0.036,36.

As will be seen in the above, the insurance company is balancing off two contracts, one against the other. Each of the insured persons is bound to pay the insurance company a definite sum of money now and at the beginning of each following year that he is alive. The aggregate amount of money that any one man will pay the company is indeterminate—some will pay more, and others less. In return for these premiums payable by the insured, the insurance company makes its promise to pay \$1,000 at the close of the year in which the insured dies. The present value of this payment is also indeterminate, for we must consider the factor of discount. If the death of an insured person occurs soon, his payments to the insurance company will be very small, and the payment by the insurance company will have a present value of nearly the full \$1,000. Thus it is only by making similar contracts with a large number of persons that the insurance company can afford to make the contracts at all.

Tables of annual premiums during life for \$1,000 insurance at various ages and on the basis of different mortality tables and

rates of interest will be found among the tables at the back of this book: also tables showing the present values of an annuity-due.

WHOLE LIFE INSURANCE WITH PREMIUMS PAYABLE FOR A TERM OF YEARS ONLY:—In the foregoing paragraphs we have examined the method of finding the premium to be paid each year of a man's life, to buy insurance for life. It is often desired to have the premium payments limited to a term of years instead of falling due each year of life. The single premium for the insurance to be given is precisely the same as where premiums are payable annually, for the insurance given is the same. The following table gives us the basis for calculating the proper annual premium to be paid for a period of 20 years only. The American Table with 3 per cent. interest, and an insured person 50 years old, are assumed as before. An explanation follows the table.

*The Calculation of the Present Value of \$1 Paid Now and at the Beginning of Each Year for 19 Years further During the Life of a Man Aged 50, Based on the American Experience Table and 3 Per Cent. Interest.*

YEAR.	Attained age at beginning of year.	(1) Present value of \$1 to be paid certainly at beginning of years indicated.	(2) Tabular number living at beginning of each year.	Product of (1) × (2). Present value of sum to be received by company in each year.
1	50	\$1.000 000	69 804	\$69 804.000 000
2	51	0.970 874	68 842	66 836.907 908
3	52	.942 596	67 841	63 946.655 236
4	53	.915 142	66 797	61 128.740 174
5	54	.888 487	65 706	58 378.926 822
6	55	.862 609	64 563	55 692.624 867
7	56	.837 484	63 364	53 066.336 176
8	57	.813 092	62 104	50 496.265 568
9	58	.789 409	60 779	47 979.489 611
10	59	.766 417	59 385	45 513.673 545
11	60	.744 094	57 917	43 095.692 198
12	61	.722 421	56 371	40 723.594 191
13	62	.701 380	54 743	38 395.645 340
14	63	.680 951	53 030	36 110.831 530
15	64	.661 118	51 230	33 869.075 140
16	65	.641 862	49 341	31 670.112 942
17	66	.623 167	47 361	29 513.812 287
18	67	.605 016	45 291	27 401.779 656
19	68	.587 395	43 133	25 336.108 535
20	69	.570 286	40 890	23 318.994 540

Total Present Value of Sums Receivable.....\$902 279.266 266

Total Present Value as above	\$902,279.266,266	=	\$12.926
Tabular Number Living at Age 50	69,804		

As will be seen by comparison with the previous table, this calculation is exactly the same as that, except that the payments for only 20 years are included in the sum of present values. As before, the present value of each person's payments, \$12.926, is found by dividing \$902,279.266,266, the total sum of present values, by 69,804, the number of persons in the group. As the payments of \$1 to the company continue for such a short period, they are worth to the company considerably less than the payments throughout life: and, if the amount of premium to be paid by

the insured is to be measured by this limited series of payments, the amount to be paid yearly, i.e. the annual premium, will therefore be greater. As before, we find the annual premium payable for 20 years by dividing the single premium by the present value of the series of possible 20 annual payments:— $\$555.215 \div \$12.926 = \$42.95$ . Tables of annual premiums limited to various terms of years will be found in the latter part of the book.

TERM INSURANCE, PURE ENDOWMENT, AND THEIR COMBINATION, ENDOWMENT INSURANCE:—At the close of our investigation of the method of computing the net single premium for whole life insurance we saw what the effect on the calculation would be if the insurance were made to cease at the close of a term of, say, 20 years. The single premium for such an insurance for \$1,000 at age 50, on the basis of the mortality table and rate of interest heretofore assumed, was found to be \$317.60. We have also just found how to value payments by the insured limited to a period of 20 years, so that we are in position to see what annual premium would be the equivalent of the single premium for this term insurance. As in other cases, we divide the single premium, \$317.60, by the present value of the annual payments of \$1, \$12.926, and have as a result \$24.57.

We now turn for a time from insurance proper, and take up what is called "Pure Endowment." This is the name given to a contract by which an insurance company in consideration of a single or an annual premium binds itself to pay a sum of money to a man at some future date, provided he then be living. It is seldom issued except in combination with a term insurance contract. The calculation is simple, and we will use some of the data before employed in connection with the calculation of an annual premium. The question then is:—"For what sum paid now by X, one of 69,804 persons 50 years old, can an insurance company guarantee to pay X on his 70th birthday the sum of \$1?"

The mortality table shows us that of the 69,804 persons living at age 50, only 38,569 will be living 20 years later, at the age of 70. Therefore, if the company made such an agreement with each of the 69,804, it would be certainly liable for \$38,569, 20 years hence. This sum discounted at 3 per cent. for 20 years has a present value of  $38,569 \times \$0.553,676 = \$21,354.729,644$ , which is the sum the company must collect from the group. As in other



cases, each should pay an equal share of this sum, which is  $\$21,354.729,644 \div 69,804 = \$0.305,92$ , the single premium for \$1, 20-year pure endowment. For \$1,000 the single premium would be \$305.92, or a thousand times as much.

Such a contract is generally paid for by a series of premiums for a term of years, or until the payment by the company becomes due. Here we wish to know what premium paid for 20 years will be equivalent to the single premium for \$1,000 Pure Endowment, \$305.92. The value to the company of an annuity at age 50 of \$1 now and at the beginning of each of the following 19 years of life was found to be \$12.926. In accordance with the rule as to annual premiums we have  $\$305.92 \div \$12.926 = \$23.67$ , which is the net annual premium for the contract.

ENDOWMENT INSURANCE, OR "ENDOWMENT":—Policies are often issued guaranteeing a payment of \$1,000 at the death of the insured if he die within, say, 20 years, and also providing that if he survive the 20 years the \$1,000 shall be paid to him. Obviously we have in the two policies just described the means of forming this latter one. The Term Policy guarantees a payment of the sum insured only in case the insured die within 20 years. The Pure Endowment guarantees that only if he lives 20 years will he receive that sum. If a man takes out both policies, or, what is really done, takes out a policy which combines both elements, he should pay both premiums. Thus, the net annual premium for \$1,000 20-year endowment insurance at age 50 would be  $\$24.57 + \$23.67 = \$48.24$ .

Tables of premiums for this form of contract for various periods, etc., are to be found among the tables at the end of the book.

OTHER PLANS:—We have now described most of the leading forms of life insurance contract. Any other peculiar policy issued by a company is based on the same general principles. One of these somewhat commonly used, known as the "Return Premium Policy," is discussed later, as its clear description would be impracticable until the student had followed through the algebraic demonstration which follows in Chapter VI. Various forms of Annuity Policy are described in Chapter IX.



## CHAPTER IV.

## NET RESERVES.

WHEN an insurance company receives from a man 50 years old \$555.215 (see tables) as a single premium for \$1,000 insurance for life, it must be prepared to pay his representatives that \$1,000 from the funds received from him and others likewise insured, at any time. According to theory, as based on the American Experience Table, it will certainly have done so within 46 years, or before he becomes 96 years old. As it has received from the insured all that it is going to receive, it must therefore hold on his account sufficient funds to meet the obligation. We now make a calculation to prove the sufficiency of this single premium, and find the amount that the company must hold in successive years. For simplicity and brevity we will use in illustration the same age and number of persons that were used in the original calculation of the net single premium, and go through the history of the group until all have died. The amount of insurance is taken as \$1, so as to avoid unduly large amounts.

RESERVES ON WHOLE LIFE POLICIES PAID FOR BY SINGLE PREMIUMS:—We assume that the company receives, at the same date, from each of a group of 69,804 persons 50 years old (the number at that age in the mortality table) insured for \$1, the sum of \$0.555,215, or in the aggregate \$38,756.228; the mortality will occur according to the American Experience Table, and money placed at interest earns 3 per cent. per annum. As each man is insured for \$1, the death claims payable each year will be \$1 for each death in that year, according to the mortality table. The headings of the columns describe the increments and decrements made in the fund, and an analysis of the table follows.

*Table based on American Experience Table and 3 per cent. interest the amounts which must be on hand at the end of successive insured group of 69,804 persons 50 years old and their survivors.*

Year of insurance.	Attained age at beginning of year.	(1) Tabular number living at beginning of year. Number insured.	(2) Amount on hand at beginning of year for group and survivors.	(3) Three per cent. interest earned in year on sum in (2).
1	50	69 804	\$38 756.228	\$1 162.687
2	51	68 842	38 956.915	1 168.707
3	52	67 841	39 124.622	1 173.739
4	53	66 797	39 254.361	1 177.631
5	54	65 706	39 340.992	1 180.230
6	55	64 563	39 378.222	1 181.347
7	56	63 364	39 360.569	1 180.817
8	57	62 104	39 281.386	1 178.442
9	58	60 779	39 134.828	1 174.045
10	59	59 385	38 914.873	1 167.446
11	60	57 917	38 614.319	1 158.430
12	61	56 371	38 226.749	1 146.802
13	62	54 743	37 745.551	1 132.367
14	63	53 030	37 164.918	1 114.948
15	64	51 230	36 479.866	1 094.396
16	65	49 341	35 685.262	1 070.558
17	66	47 361	34 775.820	1 043.275
18	67	45 291	33 749.095	1 012.473
19	68	43 133	32 603.568	978.107
20	69	40 890	31 338.675	940.160
21	70	38 569	29 957.835	898.735
22	71	36 178	28 465.570	853.967
23	72	33 730	26 871.537	806.146
24	73	31 243	25 190.683	755.720
25	74	28 738	23 441.403	703.242
26	75	26 237	21 643.645	649.309
27	76	23 761	19 816.954	594.509
28	77	21 330	17 980.463	539.414
29	78	18 961	16 150.877	484.526
30	79	16 670	14 344.403	430.332
31	80	14 474	12 578.735	377.362
32	81	12 383	10 865.097	325.953
33	82	10 419	9 227.050	276.812
34	83	8 603	7 687.862	230.636
35	84	6 955	6 270.498	188.115

*est, to prove the sufficiency of the net single premium, and to show  
 ance years; to provide for a whole life insurance of \$1 each on a*

(4)  (2)+(3) Sum of principal and interest.	(5)  Death claims by Mortality Table due at end of year. (Deduct).	(6)  (4) — (5) Amount on hand end of year after payment of death claims. (Aggregate re- serve).	Attained age at end of year.	(7)  (6) + [next line of (1)]. Amount held for each survivor of the year. (Reserve for in- dividual \$1 in- surance.)
\$39 918.915	\$962	\$38 956.915	51	\$0.565 89
40 125.622	1 001	39 124.622	52	.576 71
40 298.361	1 044	39 254.361	53	.587 67
40 431.992	1 091	39 340.992	54	.598 74
40 521.222	1 143	39 378.222	55	.609 92
40 559.569	1 199	39 360.569	56	.621 18
40 541.386	1 260	39 281.386	57	.632 51
40 459.828	1 325	39 134.828	58	.643 89
40 308.873	1 394	38 914.873	59	.655 30
40 082.319	1 468	38 614.319	60	.666 72
39 772.749	1 546	38 226.749	61	.678 13
39 373.551	1 628	37 745.551	62	.689 51
38 877.918	1 713	37 164.918	63	.700 83
38 279.866	1 800	36 479.866	64	.712 08
37 574.262	1 889	35 685.262	65	.723 24
36 755.820	1 980	34 775.820	66	.734 27
35 819.095	2 070	33 749.095	67	.745 16
34 761.568	2 158	32 603.568	68	.755 89
33 581.675	2 243	31 338.675	69	.766 41
32 278.835	2 321	29 957.835	70	.776 73
30 856.570	2 391	28 465.570	71	.786 82
29 319.537	2 448	26 871.537	72	.796 67
27 677.683	2 487	25 190.683	73	.806 28
25 946.403	2 505	23 441.403	74	.815 69
24 144.645	2 501	21 643.645	75	.824 93
22 292.954	2 476	19 816.954	76	.834 01
20 411.463	2 431	17 980.463	77	.842 97
18 519.877	2 369	16 150.877	78	.851 79
16 635.403	2 291	14 344.403	79	.860 49
14 774.735	2 196	12 578.735	80	.869 06
12 956.097	2 091	10 865.097	81	.877 42
11 191.050	1 964	9 227.050	82	.885 60
9 503.862	1 816	7 687.862	83	.893 63
7 918.498	1 648	6 270.498	84	.901 58
6 458.613	1 470	4 988.613	85	.909 50

*Table based on American Experience Table and 3 per cent. interest*

Year of insurance.	Attained age at beginning of year.	(1) Tabular number living at beginning of year. Number insured.	(2) Amount on hand at beginning of year for group and survivors.	(3) Three per cent. interest earned in year on sum in (2).
36	85	5 485	\$4 988.613	\$149.658
37	86	4 193	3 846.271	115.388
38	87	3 079	2 847.659	85.430
39	88	2 146	2 000.089	60.003
40	89	1 402	1 316.092	39.483
41	90	847	800.575	24.017
42	91	462	439.592	13.188
43	92	216	206.780	6.203
44	93	79	75.983	2.279
45	94	21	20.262	.608
46	95	3	2.870	.086

*to prove the sufficiency of the net single premium, etc.—continued.*

(4) (2) + (3) Sum of principal and interest.	(5) Death claims by Mortality Table due at end of year. (Deduct).	(6) (4) — (5) Amount on hand end of year after payment of death claims. (Aggregate re- serve).	Attained age at end of year.	(7) (6) + [next line of (1)]. Amount held for each survivor of the year. (Reserve for in- dividual \$1 in- surance.)
\$5 138.271	\$1 292	\$3 846.271	86	\$0.917 31
3 961.659	1 114	2 847.659	87	.924 86
2 933.089	933	2 000.089	88	.932 01
2 060.092	744	1 316.092	89	.938 72
1 355.575	555	800.575	90	.945 19
824.592	385	439.592	91	.951 50
452.780	246	206.780	92	.957 31
212.983	137	75.983	93	.961 81
78.262	58	20.262	94	.964 86
20.870	18	2.870	95	.956 67
2.956	3	-0.044	..	.....



The 3 per cent. interest (\$1,162.687) earned the first year on the aggregate single premium (\$38,756.228) is added to that premium. Then the death claims \$962, which according to theory would fall due, are deducted. The remainder (\$38,956.915) will then be on hand at the beginning of the second year. Three per cent. interest on this sum (\$1,168.707) is added to it and the death claims of the year (\$1,001) are deducted, leaving \$39,124.622 to be carried forward to the beginning of the third year. This operation is repeated until the last claim has been paid. When the right amount to start with, and the right deduction to be made each year, are understood, the table above presents little difficulty. It is simply a matter of seeing that the company is getting 3 per cent. interest on all the funds it retains from year to year. The sum on hand at the beginning of the 46th year should be just enough with interest to cover the \$3 of death claims falling due at the close of that year. The slight discrepancy is due to enforced limitation in the number of decimal places used, which also affects the amounts in the last column.

Examining the table a little further we note that as 962 persons have died in the first year, the \$38,957 remaining at the end of the first year—or the beginning of the second year—is held on account of only 68,842 persons who are now 51 years old. If we refer to the table of Single Premiums for Whole Life Insurance, on the American Table with 3 per cent. interest, and take that at age 51, we will find that it is (for \$1 insurance instead of \$1,000 insurance) the sum, \$0.565,89. Now if 68,842 persons, 51 years old, wished to come into the company at this point they would each have to pay this amount, and in the aggregate would pay \$38,957, which would be the sum necessary to meet all obligations under these new contracts. It will be seen immediately that this aggregate premium is exactly the sum of money which the company would have on hand on account of the 68,842 survivors of the original group of 69,804 persons that had entered the company a year previous. Therefore the single premium collected from the group of insured persons was just sufficient to meet the death claims of the first year and leave on hand a sum which is equivalent to the aggregate single premium for the survivors now one year older, or 51 years of age. For the individual policy, as shown in the last column, the sum reserved or new single premium, is \$0.565,89.

In the same way the sum of money, \$38,614, held by the company at the close of the 10th year (or the beginning of the 11th) is the same as the aggregate single premium for 57,917 persons 60 years old entering the company at that time. That is,  $57,917 \times \$0.666,72 = \$38,614$ . The table continues until all who entered have died.

Each year the company is called upon for an ascertained sum, and at the close of that year has no more than just enough money on hand to meet the death claims which will fall due in later years. Then if the company holds, for any reason, even a dollar less than the amount indicated, it will to that extent have less than enough to meet its full obligations. According to our theory, therefore, the company, to be solvent, must hold the amount stated in the table at the various times on account of this group of persons, independently of any other obligations it may have contracted.

This sum which the company must hold to ensure its solvency is known as the "*Net Reserve*." As will be supposed, it varies in amount with the form of the contract, age at entry, and the number of years in force. The particular form of reserve above indicated is the simplest of any in a case where insurance is given for the whole of life.

**RESERVE FOR WHOLE LIFE INSURANCE PAID FOR BY ANNUAL PREMIUMS:**—We will now assume that the 69,804 persons who came into the company together at the age of 50 agreed to pay, for the same whole life insurance of \$1, premiums at the beginning of each year of their lives, instead of the one lump sum when the contract was entered into. Referring to page 28, we will see that the net annual premium that each would thus have to pay would be \$0.036,36. (More accurate calculation gives us the figures \$0.036,3576.) The sum which the group would pay the first year would be  $69,804 \times \$0.036,3576 = \$2,537.906$ . The deaths in this case and the amounts payable at death would be exactly the same as in the case of the single premium policies just described. A table showing how the account would run, from the beginning to the end of the longest existing contract, is shown on the ensuing pages and an explanation follows.

*Table based on American Experience Table and 3 per cent. interest amounts must be on hand at the end of successive insurance years, Life Insurance of \$1 each on a group of 69,804 persons 50 years*

Year of insurance.	Attained age at beginning of year.	(1) Tabular number living at beginning of year. Number insured.	(2) Sum on hand end of previous year. See (8).	(3) Annual premiums paid at beginning of year. (1) × \$0.036,3576	(4) Total on hand at beginning of year.
1	50	69 804	.....	\$2 537.906	\$2 537.906
2	51	68 842	\$1 652.043	2 502.930	4 154.973
3	52	67 841	3 278.622	2 466.536	5 745.158
4	53	66 797	4 873.513	2 428.579	7 302.092
5	54	65 706	6 430.155	2 388.912	8 819.067
6	55	64 563	7 940.639	2 347.356	10 287.995
7	56	63 364	9 397.635	2 303.763	11 701.398
8	57	62 104	10 792.440	2 257.952	13 050.392
9	58	60 779	12 116.904	2 209.779	14 326.683
10	59	59 385	13 362.483	2 159.096	15 521.579
11	60	57 917	14 519.226	2 105.723	16 624.949
12	61	56 371	15 577.697	2 049.514	17 627.211
13	62	54 743	16 528.027	1 990.324	18 518.351
14	63	53 030	17 360.902	1 928.044	19 288.946
15	64	51 230	18 067.614	1 862.600	19 930.214
16	65	49 341	18 639.120	1 793.920	20 433.040
17	66	47 361	19 066.031	1 721.932	20 787.963
18	67	45 291	19 341.602	1 646.672	20 988.274
19	68	43 133	19 459.922	1 568.212	21 028.134
20	69	40 890	19 415.978	1 486.662	20 902.640
21	70	38 569	19 208.719	1 402.276	20 610.995
22	71	36 178	18 838.325	1 315.345	20 153.670
23	72	33 730	18 310.280	1 226.342	19 536.622
24	73	31 243	17 635.721	1 135.920	18 771.641
25	74	28 738	16 829.790	1 044.845	17 874.635
26	75	26 237	15 909.874	953.914	16 863.788
27	76	23 761	14 893.702	863.893	15 757.595
28	77	21 330	13 799.323	775.508	14 574.831
29	78	18 961	12 643.076	689.376	13 332.452
30	79	16 670	11 441.426	606.081	12 047.507
31	80	14 474	10 212.932	526.240	10 739.172
32	81	12 383	8 970.347	450.216	9 420.563
33	82	10 419	7 739.180	378.810	8 117.990
34	83	8 603	6 545.530	312.784	6 858.314
35	84	6 955	5 416.063	252.867	5 668.930



*proving the sufficiency of annual premiums, and showing what in addition to the premiums yet to come due, to provide for a Whole old and their survivors.*

(5) Three per cent. in- terest on sum in (4).	(6) (4) + (5) Sum of principal and interest.	(7) Death claims by Mortality Table due at end of year. (Deduct).	(8) (6) — (7) Amount on hand end of year after payment of death claims. (Aggregate reserve).	Year of insurance.	(9) (8) + [next line of (1)]. Amount held for each survivor of the year. (Reserve for individual \$1 insurance.)
\$76.137	\$2 614.043	\$962	\$1 652.043	1	\$0.024 00
124.649	4 279.622	1 001	3 278.622	2	.048 33
172.355	5 917.513	1 044	4 873.513	3	.072 96
219.063	7 521.155	1 091	6 430.155	4	.097 86
264.572	9 083.639	1 143	7 940.639	5	.122 99
308.640	10 596.635	1 199	9 397.635	6	.148 31
351.042	12 052.440	1 260	10 792.440	7	.173 78
391.512	13 441.904	1 325	12 116.904	8	.199 36
429.800	14 756.483	1 394	13 362.483	9	.225 01
465.647	15 987.226	1 468	14 519.226	10	.250 69
498.748	17 123.697	1 546	15 577.697	11	.276 34
528.816	18 156.027	1 628	16 528.027	12	.301 92
555.551	19 073.902	1 713	17 360.902	13	.327 38
578.668	19 867.614	1 800	18 067.614	14	.352 68
597.906	20 528.120	1 889	18 639.120	15	.377 76
612.991	21 046.031	1 980	19 066.031	16	.402 57
623.639	21 411.602	2 070	19 341.602	17	.427 05
629.648	21 617.922	2 158	19 459.922	18	.451 16
630.844	21 658.978	2 243	19 415.978	19	.474 83
627.079	21 529.719	2 321	19 208.719	20	.498 04
618.330	21 229.325	2 391	18 838.325	21	.520 71
604.610	20 758.280	2 448	18 310.280	22	.542 85
586.099	20 122.721	2 487	17 635.721	23	.564 47
563.149	19 334.790	2 505	16 829.790	24	.585 63
536.239	18 410.874	2 501	15 909.874	25	.606 39
505.914	17 369.702	2 476	14 893.702	26	.626 81
472.728	16 230.323	2 431	13 799.323	27	.646 94
437.245	15 012.076	2 369	12 643.076	28	.660 79
399.974	13 732.426	2 291	11 441.426	29	.686 35
361.425	12 408.932	2 196	10 212.932	30	.705 61
322.175	11 061.347	2 091	8 970.347	31	.724 41
282.617	9 703.180	1 964	7 739.180	32	.742 79
243.540	8 361.530	1 816	6 545.530	33	.760 84
205.749	7 064.063	1 648	5 416.063	34	.778 73
170.068	5 838.998	1 470	4 368.998	35	.796 54



*Table based on American Experience Table and 3 per cent. inter*

Year of insurance.	Attained age at beginning of year.	(1) Tabular number living at beginning of year. Number insured.	(2) Sum on hand end of previous year. See (8).	(3) Annual premiums paid at beginning of year. (1) $\times$ \$0.036,3576	(4) Total on hand at beginning of year.
36	85	5 485	\$4 368.998	\$199.421	\$4 568.419
37	86	4 193	3 413.472	152.447	3 565.919
38	87	3 079	2 558.897	111.945	2 670.842
39	88	2 146	1 817.967	78.023	1 895.990
40	89	1 402	1 208.870	50.973	1 259.843
41	90	847	742.638	30.795	773.433
42	91	462	411.636	16.797	428.433
43	92	216	195.286	7.853	203.139
44	93	79	72.233	2.872	75.105
45	94	21	19.358	0.764	20.122
46	95	3	2.726	0.109	2.835

*est proving the sufficiency of annual premiums, etc.—continued.*

(5)	(6)	(7)	(8)		(9)
Three per cent. in- terest on sum in (4).	(4) + (5) Sum of principal and interest.	Death claims by Mortality Table due at end of year. (Deduct).	(6)—(7) Amount on hand end of year after payment of death claims. (Aggregate reserve).	Year of insurance.	(8) + [next line of (1)]. Amount held for each survivor of the year. (Reserve for individual \$1 insurance.)
\$137.053	\$4 705.472	\$1 292	\$3 413.472	36	\$0.814 09
106.978	3 672.897	1 114	2 558.897	37	.831 08
80.125	2 750.967	933	1 817.967	38	.847 14
56.880	1 952.870	744	1 208.870	39	.862 25
37.795	1 297.638	555	742.638	40	.876 79
23.203	796.636	385	411.636	41	.890 99
12.853	441.286	246	195.286	42	.904 10
6.094	209.233	137	72.233	43	.914 34
2.253	77.358	58	19.358	44	.921 81
0.604	20.726	18	2.726	45	.908 67
0.085	2.920	3	— 0.080	46	.....

The account runs as follows:—\$2,537.906 is received from the group as the aggregate first premium, and to this is added the 3 per cent. interest earned in the year (\$76.137). Then deduction is made of the \$962 of death claims falling due at the close of the year. The balance (\$1,652.043) is retained and carried forward to the beginning of the second year. At that time there remain 68,842 insured persons each of whom pays the second premium of \$0.036,3576, the aggregate premium being \$2,502.930. These new premiums are then added to the sum retained from the previous year, 3 per cent. interest on the whole (\$124.649) is added, and the death claims of the year (\$1,001) deducted, leaving the balance (\$3,278.622) to be carried forward to the next year. This series of operations is repeated each year following, the amount of annual premiums received becoming less and less as the number of insured persons decreases. At the end of the last, or 46th year, there will theoretically be just enough money left to pay the \$3 of death claims. The slight discrepancy is due to the small number of decimal places used in this table and does not affect the principle involved.

In the foregoing illustration the company has to pay out just as much in death claims as in the previous case, but it collects at the outset only a small fraction of the amount of premium taken in the case of the single premium policies. It has, however, in this case the insured's obligation to pay premiums at the beginning of each year of life. The practical possibility that the insured may cease to pay premiums for some other reason than death will be discussed separately, and be shown not to affect the validity of the principle.

Taking now the condition of things at the close of the first year of these policies' history, we find that the amount on hand is \$1,652. The corresponding amount held at the end of the first year for the single premium contracts was \$38,957, or \$37,305 greater than in the latter case. How then can the insurance company be solvent? This difficulty is obviated by the company's taking credit for the premiums which are yet to be collected, on account of these policies. In this case at the end of one year—or the beginning of the second—there are 68,842 persons still in the company, and each of them is bound to pay the insurance company \$0.036,3576 now and at the beginning of each year of after-life. Referring to our annuity tables on the American

Experience Table with 3 per cent. interest, we find the value at the age of 51 to be \$14.9045, which is the present value of \$1 now and at the beginning of each year hereafter during life. The value of the future premiums of \$0.036,3576 paid on those conditions would therefore be  $\$0.036,3576 \times 14.9045 = \$0.541,892$ , for each of the 68,842. The aggregate present value would be 68,842 times that sum, or \$37,305.

This last sum it will be seen is equal to the difference between the corresponding aggregate reserve in the case of the single premium policies, and the sum which according to our latter account would be on hand at the close of the first year. The figures in this case then are: \$39,957, the reserves (or single premiums at the advanced age) for the single premium policies, less \$37,305, the present value of future premiums on the annual premium policies, gives \$1,652, the amount left on hand for the remaining annual premium policies. For the individual policy for \$1 the corresponding figures are:—\$0.565,89 less ( $\$0.036,3576 \times 14.9045 =$ ) \$0.541,89 gives \$0.02400, which is the reserve indicated in the last column of the table at the close of one year for each surviving policy.

Similarly at the close of the 10th year, the amount left on hand, \$14,519, is \$24,095 less than at the corresponding time under the single premium contracts, as shown in the previous table, and this latter sum may be shown to be equal to the then present value of the aggregate annual premiums yet to be received. The figures in this case are 57,917, the number of persons, times \$11.4427, the appropriate \$1 annuity value, times \$0.036,3576, the premium; the product, disregarding decimals, being \$24,095. For the individual policy for \$1 insurance the corresponding figures are:—\$0.666,72 less ( $\$0.036,3576 \times 11.4427 =$ ) \$0.416,03, gives \$0.250,69, which is the reserve indicated, in the last column of the table, at the close of 10 years for each surviving policy.

On examining the table we will see that the total amounts on hand at the end of the successive years, which are known as the "*Terminal Reserves*," (to distinguish them from reserves at other times in the year,) will increase for a time and then decrease. The reserve for the individual policy, however, steadily increases. It is well now to consider the possibility that some persons while still living may fail to pay the premiums as they become due,



and see whether the company is warranted in taking credit for the full amount of future premiums on each policy.

Under the contract each man pays at the beginning of each year his premium of \$0.036,36. If he does not pay the premium, he will not be insured in that year. Now, if we refer to the table on page 21, and begin with age 50, we will see that the least the company could receive from the man to give him insurance for one year (for \$1 instead of \$1,000) would be \$0.013,38. Therefore, if the man should fail to pay the second annual premium on his whole life policy, there would be no deficiency on his account, but instead he would have paid \$0.022,98 more than just enough for the one year's insurance received. (Besides this, as the man had failed to pay the second annual premium, the company would be relieved from further liability to pay \$1 at his death, and so would not need to hold on his account the excess \$0.022,98 received, with its accumulations.) In the following years so long as the least premium the company could receive from him is less than the premium actually received, it is clear that the company has not incurred a risk greater than that paid for, and so has not been "out" on the policy. This is true in the case given for illustration up to age 65. When the man becomes 65, and thereafter during his life, the least premium that could be charged for one year's insurance is greater than the amount he has agreed to pay. Yet the insurance company has not taken more risk than was paid for. The fact that earlier in the policy's history the premium was greater than temporarily necessary, and that the excess in each case has been held on the man's account, takes care of this. At the time the man becomes 65, or at the end of the 15th policy year, the company is holding on account of the 49,341 survivors the sum of \$18,639. As the amount of this reserve is proportional to the number of survivors in the group, the portion held on account of any one man would be  $\frac{1}{49,341}$  of that, or \$0.377,76, as shown in the last column of the table. Therefore, at the beginning of the next year, the company would hold on account of any one man \$0.377,76, besides the premium of \$0.036,36 then paid, or in the aggregate \$0.414,12, which is far more than enough to pay for one year's insurance of \$1. The same relation may be shown to be true at any greater age attained, and the foregoing line of proof may be used in the case of any

of the forms of policy which have thus far been described, at any age of original issue.

The amounts held, therefore, according to the calculation, at the end of each year, are the "reserves" at those dates, and we are now able to formulate a rule for their calculation at any time without going through the account from the issue of the policy. The rule is as follows: "The net reserve, or 'net value,' at the end of any year, is to be found by deducting from the net single premium for the insurance, at the attained age of the insured, the then present value of the net premiums yet to be received. The balance will be the net reserve."

Further demonstration of this rule, and its application to all forms of policy, will be given after the student has mastered the algebraic symbols and formulas in the chapters following, and thus become able to carry on the calculations in a less cumbersome way.

It may, however, be well at this point to observe that, when a company has outstanding many hundreds of millions of insurance which has been in force for several years, it must of necessity hold in safe investments vast amounts of money on account of those policies. Even with such immense assets the company is not however to be considered wealthy, for the funds and their accumulations must eventually be repaid to its policy-holders.

## CHAPTER V.

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### MORTALITY TABLES AND INTEREST ASSUMPTIONS.

THE foregoing discussions have, for the sake of clearness, been based on only one table of mortality, the American Experience Table, and one rate of interest, 3 per cent. The principles involved, however, were not in any way dependent on these assumptions, but would hold good for any table of mortality and rate of interest which might be taken as a basis.

There are two tables of mortality in general use in the United States. One of these, the "Actuaries" or "Combined Experience" Table, was issued in 1843 and was derived from the experience of seventeen English life insurance companies. This table, with 4 per cent. interest, is the basis of a large amount of insurance still in force, but is practically in disuse as a basis for the issue of new policies. This table is given at the close of this chapter. The Actuaries' Table supposes that the limit of life is just short of 100 years, but in other respects does not differ very radically from the American Experience Table.

The American Experience Table, formed about 1866 by Shepard Homans, was based upon the experience of the Mutual Life Insurance Company of New York with such modifications as its author thought desirable. It has been in use as the basis of insurance contracts to some extent from the time of its formation, and now, with 3 or  $3\frac{1}{2}$  per cent. interest assumed, has superseded the Actuaries' Table as a basis for the issue of new contracts.

In the matter of interest assumptions, it became evident, from our discussion of the principles of the science, that payments both by the insured and by the company are discounted for long periods of time. Therefore great care must be taken in assuming what rate of interest can certainly be obtained on invested funds throughout these long periods. For if it were assumed that money would earn a higher rate of interest than it actually can earn, there would result a deficiency in the funds from lack of sufficient interest earnings, while if too low a rate were assumed, there would result a surplus. As one of the prime requisites of



life insurance is certainty of payment of the sum insured, it is customary to assume a rate rather too low than too high, so as to have a surplus of interest rather than a deficiency. For this reason either 3 or  $3\frac{1}{2}$  per cent. interest is now assumed by the companies as a basis for the premiums on new policies, though they are on the average actually earning well over 4 per cent. interest on their invested funds, and expect to be able to do so for many years to come. The assumption of a lower rate of interest makes necessary larger net premiums and reserves than would be the case if a higher rate of interest were assumed in combination with the same mortality table.

Besides the Actuaries and American Experience tables, which are the only ones in general use in this country, there are many other mortality tables, some based like these on experience with insured lives, some on general population, and some showing the peculiarities of mortality experience in particular classes of persons. Below is given a short description of a few of these tables.

The Northampton Table, published by Dr. Price in 1783, is a "population table," based chiefly on the records of the deaths in a portion of the town of Northampton, England. The author of the table had very inaccurate data as to the number living and made some assumptions which later investigations have proved to be incorrect. The number of lives involved is also quite small. For these reasons there are many serious defects in the table, and it is never employed in life insurance calculations in this country. In many localities, however, the courts still refuse to accept anything but this very faulty table for the valuation of life interests and similar computations.

The Carlisle Table is another table based upon records of population. It was published in 1815 by Joshua Milne, and was constructed upon correct scientific principles. In spite of some irregularities, this table has proved very valuable as a basis for the calculation of complex contracts involving more than one life.

In 1869 the British Institute of Actuaries published the results of its investigation of the experience of twenty British companies. Several tables of mortality were derived from these data, the one best known being the  $H^M$  or "Healthy Males" table. This has long been in use in Canada with different rates of interest for the calculation of premiums and as a basis of legal enactments relating to insurance.



*"Actuaries" or "Combined Experience" Table of Mortality.*

Age.	Number living.	Number of deaths.	Death rate per 100.	Age.	Number living.	Number of deaths.	Death rate per 100.
10	100 000	676	0.68	55	63 469	1 375	2.17
11	99 324	674	.68	56	62 094	1 436	2.31
12	98 650	672	.68	57	60 658	1 497	2.47
13	97 978	671	.68	58	59 161	1 561	2.64
14	97 307	671	.69	59	57 600	1 627	2.82
15	96 636	671	.69	60	55 973	1 698	3.03
16	95 965	672	.70	61	54 275	1 770	3.26
17	95 293	673	.71	62	52 505	1 844	3.51
18	94 620	675	.71	63	50 661	1 917	3.78
19	93 945	677	.72	64	48 744	1 990	4.08
20	93 268	680	.73	65	46 754	2 061	4.41
21	92 588	683	.74	66	44 693	2 128	4.76
22	91 905	686	.75	67	42 565	2 191	5.15
23	91 219	690	.76	68	40 374	2 246	5.56
24	90 529	694	.77	69	38 128	2 291	6.01
25	89 835	698	.78	70	35 837	2 327	6.49
26	89 137	703	.79	71	33 510	2 351	7.02
27	88 434	708	.80	72	31 159	2 362	7.58
28	87 726	714	.81	73	28 797	2 358	8.19
29	87 012	720	.83	74	26 439	2 339	8.85
30	86 292	727	.84	75	24 100	2 303	9.56
31	85 565	734	.86	76	21 797	2 249	10.32
32	84 831	742	.87	77	19 548	2 179	11.15
33	84 089	750	.89	78	17 369	2 092	12.04
34	83 339	758	.91	79	15 277	1 987	13.01
35	82 581	767	.93	80	13 290	1 866	14.04
36	81 814	776	.95	81	11 424	1 730	15.14
37	81 038	785	.97	82	9 694	1 582	16.32
38	80 253	795	.99	83	8 112	1 427	17.59
39	79 458	805	1.01	84	6 685	1 268	18.97
40	78 653	815	1.04	85	5 417	1 111	20.51
41	77 838	826	1.06	86	4 306	958	22.25
42	77 012	839	1.09	87	3 348	811	24.22
43	76 173	857	1.13	88	2 537	673	26.53
44	75 316	881	1.17	89	1 864	545	29.24
45	74 435	909	1.22	90	1 319	427	32.37
46	73 526	944	1.28	91	892	322	36.10
47	72 582	981	1.35	92	570	231	40.53
48	71 601	1 021	1.43	93	339	155	45.72
49	70 580	1 063	1.51	94	184	95	51.63
50	69 517	1 108	1.59	95	89	52	58.43
51	68 409	1 156	1.69	96	37	24	64.86
52	67 253	1 207	1.79	97	13	9	69.23
53	66 046	1 261	1.91	98	4	3	75.00
54	64 785	1 316	2.03	99	1	1	100.00

## CHAPTER VI.

## ELEMENTARY FORMULAS AND THE COMMUTATION COLUMNS.

WE now establish in the shape of algebraic formulas the general principles which have been worked out in particular instances in the foregoing chapters.

INTEREST AND DISCOUNT SYMBOLS:—In Chapter II, we found that \$1, when invested at 3 per cent. compound interest, becomes \$1.03 at the end of one year, \$1.0609 at the end of two years, \$1.0927 at the end of three years, etc. Also we learned that \$1 paid now is, under those circumstances, the equivalent of any one of those sums at the end of the period designated. Arithmetically stated, we have the foregoing as follows:  $\$1 \times 1.03 = \$1.03$ :  $\$1 \times 1.03 \times 1.03 = \$1.0609$ :  $\$1 \times 1.03 \times 1.03 \times 1.03 = \$1.0927$ :  $\$1 \times 1.03 \times 1.03 \times 1.03 \times 1.03 = \$1.1255$ , and so on. Examining this we see that, when the period is one year, the factor 1.03 occurs once; when two years, it occurs twice; when three years, it occurs three times; etc., so that, if we assume a number of years,  $n$ , and then compound \$1 yearly for that time, our calculation would be: \$1 multiplied by 1.03  $n$  times, or \$1 multiplied by the  $n$ th power of 1.03, or  $\$1 \times (1.03)^n$ , the result depending solely on what value we give to  $n$ . If \$2 is to be improved at interest each \$1 of the \$2 will obviously increase as above indicated, the result being just twice as great, and the result for any amount may be found by thus multiplying the figures for \$1 by that amount.

The above is true in principle for any rate of interest we care to assume. If 4 per cent were taken, the factor would be 1.04, that is, "one and four hundredths" instead of "one and three hundredths." We can therefore use a general symbol " $i$ " to express the rate of interest per cent., or the number of "hundredths" in the factor, and we then have in a general form " $1+i$ " as a factor. Then  $\$1 \times (1+i)$  is the amount of \$1 at  $i$  rate of interest per cent. at the end of one year;  $\$1 \times (1+i) \times (1+i)$  or  $\$1 \times (1+i)^2$ , is the amount at the end of two years; and, in

general,  $\$1 \times (1+i)^n$ , or more simply  $\$(1+i)^n$ , is the amount of \$1 at the end of  $n$  years; other amounts than \$1 being in proportion.

In the same way we may express in symbols the present values of the unit payable at periods in the future. If \$1 becomes \$1.03 at the end of one year,  $\frac{\$1}{1.03}$  is the present value, or "P. V." of \$1 payable one year hence. If \$1 becomes \$1.0609 at the end of two years,  $\frac{\$1}{1.0609}$  is the P. V. of \$1 payable two years hence. Similarly,  $\frac{\$1}{1.0927}$  is the P. V. of \$1 payable three years hence. But  $1.0609 = 1.03 \times 1.03$ , and  $1.0927 = 1.03 \times 1.03 \times 1.03$ , or in symbols,  $(1+i)^2$  and  $(1+i)^3$ . Therefore the P. V. of \$1 payable one year hence, interest being  $i$  per cent., is  $\frac{\$1}{1+i}$ ; \$1 payable two years hence is  $\frac{\$1}{(1+i)^2}$ , or what is the same thing  $\$(\frac{1}{1+i})^2$ ; and \$1 payable three years hence is  $\$(\frac{1}{1+i})^3$ ; or in general \$1 payable  $n$  years hence is  $\$(\frac{1}{1+i})^n$ , in which we can give  $i$  and  $n$  any values we choose. The present values of sums other than \$1 will be in proportion.

Another interest symbol proves specially useful in the statement of formulas. The above fraction  $\frac{1}{1+i}$  is rather cumbersome, and instead, the term  $v$  is used. Thus,  $v = \frac{1}{1+i}$ ,  $v^2 = \frac{1}{(1+i)^2}$ , or  $(\frac{1}{1+i})^2$ , and generally,  $v^n = (\frac{1}{1+i})^n$ . Here the value of  $v$  depends directly on the value which is assigned to  $i$  for the time being.

**LIFE INSURANCE SYMBOLS:**—As in the case of interest and discount, it is of great practical convenience to be able to express in symbols general relations which refer to the mortality table, and we will adopt the following simple system of notation.

Let  $l$  = the number of living at a particular age of a mortality table. The age taken is denoted by a subscript. Thus in the American Experience Table,  $l_{20} = 92,637$ .

Then in general,  $l_x$  = the number living at age  $x$ , the  $x$  being any particular age assumed for the calculation, and not an "unknown quantity," as that term is generally used in algebra.



It is often desirable to designate a value of  $l$  for an age a definite number of years greater than age  $x$ , without first assuming a value for  $x$ . Thus for an age 10 years greater than  $x$  the symbol would read  $l_{x+10}$ . If here  $x=20$ ,  $(x+10)=30$ , and  $l_{x+10}=l_{30}$ , which is 85,441 by the American Experience Table. More generally stated,  $l_{x+n}$ =the number living at an age an assumed number of years,  $n$ , greater than an assumed age  $x$ .

Let  $d$ =the number of persons dying according to a mortality table within one year after attaining a designated age. Here the subscripts are used in the same way as with the  $l$ . Thus  $d_{20}$ =the number dying in one year after attaining age 20, and by the American Experience Table  $d_{20}=723$ . The general term  $d_x$ =the number dying in one year after attaining age  $x$ , and  $d_{x+n}$  is the corresponding expression for age  $x+n$ .

NET SINGLE PREMIUMS:—We are now able by the foregoing symbols to express in general terms the principles enunciated in the previous chapters. (As the value will always be given for \$1 insurance, the dollar sign may be understood as assumed, and will be omitted throughout). Thus the premium for one year's insurance at age  $x$  will have the value  $v\frac{d_x}{l_x}$ . That means that the sum  $d_x$  will, by the mortality table, become due one year hence, on account of the  $d_x$  deaths which will occur in the first year among an assumed group of  $l_x$  persons aged  $x$ . This sum being payable a year hence must be discounted at some assumed rate, which is done by multiplying it by  $v$ , (i. e.  $\frac{1}{1+i}$ ), and this present value,  $vd_x$ , must be collected in equal parts from each of the  $l_x$  living at age  $x$ . The formula is true for any assumed age  $x$ .

SINGLE PREMIUM FOR WHOLE LIFE INSURANCE:—The single premium for whole life insurance is found by merely continuing the above formula for one year's insurance, and taking care of the payments at the deaths which will fall in later years.

For the first year's insurance we have  $\frac{vd_x}{l_x}$ , as above. For the second year we have  $\frac{v^2d_{x+1}}{l_x}$ . Here the sum of  $d_{x+1}$  will be payable because of  $d_{x+1}$  deaths from among the  $l_{x+1}$  left from the original  $l_x$  persons. This sum will fall due at the end of two years from the present, so is discounted for two years by multiplying it by  $v^2$ ,



and the present value,  $v^2d_{x+1}$ , must be assessed equally on each of the original  $l_x$  persons. Similarly for the third year, the value of each person's share of the future death cost is  $\frac{v^3d_{x+2}}{l_x}$ , for the fourth year  $\frac{v^4d_{x+3}}{l_x}$ , and in general, for the  $n$ th year  $\frac{v^nd_{x+n-1}}{l_x}$ , though, of course, if a value is assumed for  $n$  such that  $x+n-1$  would give an age greater than the highest age of the living shown in the table of mortality,  $d_{x+n-1}$  would be 0 and make the whole fraction  $\frac{v^nd_{x+n-1}}{l_x} = 0$ .

Therefore the expression for the net single premium for whole life insurance will be formed by adding the series of fractions indicated. The result is as follows:

$$A_x = \frac{vd_x}{l_x} + \frac{v^2d_{x+1}}{l_x} + \frac{v^3d_{x+2}}{l_x} + \text{etc., to table limit,}$$

$$\text{or } A_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \text{etc., to table limit,}}{l_x}$$

the symbol  $A_x$  being used to denote the value of this single premium for whole life insurance at any assumed age  $x$ .

If we were to assume that  $x=50$  we could construct from this formula the table on page 23, by reference to the mortality and interest tables. Then  $l_{50}=69,804$ ;  $v=.970,874$ ,  $d_{50}=962$  and  $vd_{50}=933.980,788$ ;  $v^2=.942,596$ ,  $d_{51}=1,001$  and  $v^2d_{51}=943.538,596$ ; and so on.

TERM INSURANCE:—If it be desired to limit the insurance to a particular period, there will be in the formula terms for only that number of years. For a five-year term insurance the formula would have the five terms,  $\frac{vd_x}{l_x}$ ,  $\frac{v^2d_{x+1}}{l_x}$ ,  $\frac{v^3d_{x+2}}{l_x}$ ,  $\frac{v^4d_{x+3}}{l_x}$ ,  $\frac{v^5d_{x+4}}{l_x}$ ; for a 10 year term insurance there would be 10 such terms; and so on, for any term we desire.

ANNUAL PREMIUMS:—To find the formula which will express in general terms the amount which paid now and at the beginning of each year of after-life will be the equivalent of the Whole Life single premium, we have to perform an operation similar to that made on page 26, where values were found arithmetically for a particular state of facts.

If  $l_x$  persons  $x$  years old contracted to pay \$1 to the insurance company now and at the beginning of each year of after-life, the amount of the first total payment would be  $l_x$ , the second,  $l_{x+1}$ ,

the third,  $l_{x+2}$ , etc. The first sum, being payable now, would be worth the full  $l_x$ ; the second,  $l_{x+1}$ , being payable one year hence, should be discounted for one year at some assumed rate of interest, and has a P. V. of  $vl_{x+1}$ ; the third payable two years hence has a P. V. of  $v^2l_{x+2}$ ; and so on for all the ages of the table following  $x$ , the sum of the series thus being:  $l_x + vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc.}$  to the limit of the table.

Now as all of the  $l_x$  persons came into this supposed contract on the same basis, the share of each, if it were to be settled for at once in a lump sum, would be that aggregate Present Value or P. V., divided by  $l_x$ . Stated algebraically this would be:

$$\frac{l_x + vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + v^4l_{x+4} + \text{etc., to table limit.}}{l_x}$$

or, as  $\frac{l_x}{l_x}$  is obviously equal to 1,

$$1 + \frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + v^4l_{x+4} + \text{etc., to table limit.}}{l_x}$$

This expression, the P. V. of an "annuity-due" at age  $x$ , is denoted by the symbol  $1+a_x$ , in which the  $a_x$  signifies the  $\frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc.}}{l_x}$ , which is the P. V. of payments of

\$1 at the end of each year of the life of a person now aged  $x$ .

ANNUAL PREMIUM FOR WHOLE LIFE INSURANCE:—We are now in a position to combine this value of a life series of \$1 payments with the single premium for whole life insurance, to find the annual premium. The single premium formula gives us the present cost of \$1 insurance for life at age  $x$ , so that we have only to divide that expression by the expression for the P. V. of a life series of \$1 payments to find how much must be paid annually in this way to be equivalent to the single premium. Algebraically stated this is:  $\frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \text{etc.}}{l_x} \div \frac{l_x + vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc.}}{l_x}$ ; the

result is  $P_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + v^4d_{x+3} + \text{etc.}}{l_x + vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc.}}$ , the expression  $P_x$  being used to denote the annual premium for an insurance at age  $x$ .

The above formula is merely an algebraic statement of general principles which were shown in Chapter III in arithmetical form for a particular age; but, though it states the principles generally, it requires nearly as many terms as in the previous arithmetical demonstration, and so does not materially shorten the labor of making a calculation.

This desirable shortening of the formulas, and saving of time, is accomplished by the use of what are known as "Commutation Tables," which will next be described. When they are understood, it will be easier to follow the demonstration of formulas for other contracts than those above shown.

COMMUTATION TABLES:—Let us take the formula for the P. V. at age  $x$  of a series of annual payments of \$1 at the beginning of each year of life, i.e. "annuity-due," and examine it. It reads as follows:

$$\frac{l_x + v^1 l_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \dots + v^n l_{x+n} + \text{etc.}, \text{ to table limit.}}{l_x}$$

If now we multiply both numerator and denominator by the quantity  $v^x$  we make no change in the fraction's value, but it reads:

$$\frac{v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + \dots + v^{x+n} l_{x+n} + \text{etc.}, \text{ to table limit.}}{v^x l_x}$$

The introduction of this common factor in numerator and denominator makes the exponent of  $v$  in every case the same as the subscript of  $l$ . The purpose of this will be shown by making use of the formula as above and ascribing values to  $x$ ,  $x+1$ ,  $x+2$ , etc. By the American Experience Table it would then appear as follows, for  $x=80$ :

$$\frac{v^{80} l_{80} + v^{81} l_{81} + v^{82} l_{82} + \dots + v^{82} l_{92} + v^{83} l_{93} + v^{84} l_{94} + v^{85} l_{95}}{v^{80} l_{80}}$$

For  $x=79$  it would be

$$\frac{v^{79} l_{79} + v^{80} l_{80} + v^{81} l_{81} + v^{82} l_{82} + \dots + v^{82} l_{92} + v^{83} l_{93} + v^{84} l_{94} + v^{85} l_{95}}{v^{79} l_{79}}$$

For  $x=50$  it would be

$$\frac{v^{50} l_{50} + v^{51} l_{51} + \dots + v^{79} l_{79} + v^{80} l_{80} + v^{81} l_{81} + v^{82} l_{82} + \dots + v^{82} l_{92} + v^{83} l_{93} + v^{84} l_{94} + v^{85} l_{95}}{v^{50} l_{50}}$$

On comparing these three instances, taken at random, it is apparent that the numerator of each contains the terms  $v^{85} l_{95}$ ,  $v^{84} l_{94}$ ,  $v^{83} l_{93}$ , and so on for every age down to the age at which the value is taken; and that the denominator, in every case, is the same as the first term in the numerator. Therefore, if the numerical values of  $v^{85} l_{95}$ ,  $v^{84} l_{94}$ , and so on down the age scale, are found for any particular mortality table (in this case the American Experience) and rate of interest, the values thus found will prove useful in computing values at any age in that table, and save much re-calculation. The result of such multiplication for



a particular age  $x$  is called  $D_x$ . That is,  $D_x = v^x l_x$ ;  $D_{30} = v^{30} l_{30}$ ;  $D_{x+n} = v^{x+n} l_{x+n}$ ; etc.

The formula for the life series of annual payments would then read for the American Experience Table:

$$D_x + D_{x+1} + D_{x+2} + \dots + D_{x+n} + D_{x+n+1} + \dots + D_{92} + \frac{D_{93} + D_{94} + D_{95}}{D_x}$$

For convenience the values of  $D_x$  are placed in a column, headed " $D_x$ ," with the assumed age  $x$  in the margin, as shown among the tables at the end of the book.

It is possible to further abbreviate the above calculation by a summation of the values of  $D_x$ . If we add  $D_{94} + D_{95}$ , we have the numerator for the above expression when  $x = 94$ . If we add to that sum  $D_{93}$  we have  $D_{93} + D_{94} + D_{95}$ , which is the corresponding numerator when  $x = 93$ ; and so on for each age younger.

The term  $N_x$  is used to express these sums. Thus,  $N_{95} = D_{95}$ ;  $N_{94} = D_{94} + D_{95}$ ;  $N_{93} = D_{93} + D_{94} + D_{95}$ ;  $N_{50} = D_{50} + D_{51} + \dots + D_{93} + D_{94} + D_{95}$ .

The values thus obtained are then usually entered in a column parallel to the " $D_x$ " column, and headed " $N_x$ ."

If then  $N_x = D_x + D_{x+1} + D_{x+2} + \dots$ , to limit of table, the above expression for the series of annual payments will take

$$\text{the form } \frac{N_x}{D_x}. \text{ Therefore } 1 + a_x = \frac{N_x}{D_x}$$

LIMITED SERIES OF ANNUAL PAYMENTS DURING LIFE:—If it be desired to find the P. V. of annual payments of \$1 during only a term of life, as  $n$  years, i.e. only  $n$  payments, the  $N_x$  table is directly useful.

Here the expression is  $\frac{l_x + v l_{x+1} + \dots + v^{n-2} l_{x+n-2} + v^{n-1} l_{x+n-1}}{l_x}$ ,

for, as the first of the  $n$  terms of the numerator is not affected by  $v$ , there can be only  $n-1$  other terms affected by  $v$  and its higher powers. When the factor  $v^x$  is introduced, this becomes

$$\frac{v^x l_x + v^{x+1} l_{x+1} + \dots + v^{x+n-2} l_{x+n-2} + v^{x+n-1} l_{x+n-1}}{v^x l_x},$$

$$\text{or } \frac{D_x + D_{x+1} + \dots + D_{x+n-2} + D_{x+n-1}}{D_x}$$

Now, by  $N_x$  we express the sum of all values of  $D_x$  beginning with the highest age of the table and ending with the  $D_x$  for the



particular age  $x$ , and by  $N_{x+n}$  we mean all such values from the highest age down to and including age  $x+n$ . It will be seen that the terms in the numerator of the above fraction are the values of  $D_x$  which are included in  $N_x$  and yet not included in  $N_{x+n}$ , for  $D_{x+n-1}$  is the next value to  $D_{x+n}$ , and  $D_{x+n}$  is included in  $N_{x+n}$ . Therefore to get the sum of  $D_x + D_{x+1} + \dots + D_{x+n-1}$ , we have only to deduct  $N_{x+n}$  from  $N_x$ , and the above fraction will then take the form  $\frac{N_x - N_{x+n}}{D_x}$ .

Similar symbols and uniform values are used for the expressions derived from the column "Number of Deaths." We have for the whole life single premium:

$$A_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + v^4d_{x+3} + \text{etc., to limit of table.}}{l_x}$$

Multiplying numerator and denominator by  $v^x$ ,

$$A_x = \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + v^{x+4}d_{x+3} + \text{etc., to limit of table.}}{v^x l_x}$$

in which  $v^x l_x = D_x$  in the denominator, and in each term of the numerator the exponent of  $v$  is greater by one than the subscript of the  $d$  of that term. Then by the American Table,

$$A_{80} = \frac{v^{81}d_{80} + v^{82}d_{81} + \dots + v^{93}d_{92} + v^{94}d_{93} + v^{95}d_{94} + v^{96}d_{95}}{v^{80}l_{80} = D_{80}}$$

$$A_{79} = \frac{v^{80}d_{79} + v^{81}d_{80} + v^{82}d_{81} + \dots + v^{93}d_{92} + v^{94}d_{93} + v^{95}d_{94} + v^{96}d_{95}}{v^{79}l_{79} = D_{79}}$$

$$A_{50} = \frac{v^{51}d_{50} + v^{52}d_{51} + \dots + v^{81}d_{80} + v^{82}d_{81} + \dots + v^{93}d_{92} + v^{94}d_{93} + v^{95}d_{94} + v^{96}d_{95}}{v^{50}l_{50} = D_{50}}$$

As the various terms of the numerators in the above cases, taken at random, are identical so far as they go, values for these terms can be calculated and tabulated for repeated use. The expression  $C_x$  is used to denote these values, the subscript  $x$  denoting the age. Therefore  $C_x = v^{x+1}d_x$ ;  $C_{30} = v^{31}d_{30}$ ;  $C_{x+n} = v^{x+n+1}d_{x+n}$ . These values are placed in a column parallel to the " $D_x$ " and " $N_x$ " columns and headed " $C_x$ ."

An operation similar to that in the case of the values of  $D_x$  is also performed with these values of  $C_x$ . Thus  $A_{95} = \frac{C_{95}}{D_{95}}$ ;  $A_{94} = \frac{C_{94} + C_{95}}{D_{94}}$ ;

$$A_{93} = \frac{C_{93} + C_{94} + C_{95}}{D_{93}}; \quad A_{80} = \frac{C_{80} + C_{81} + \dots + C_{92} + C_{93} + C_{94} + C_{95}}{D_{80}}$$

Therefore if we know the sum of the values of  $C_x$  from the highest age to the age under consideration inclusive, we can immediately

find the single premium by dividing that sum by the  $D_x$  at that age. This sum is taken by successive addition to each age, beginning at the highest age, and the results are placed in a column parallel to the " $C_x$ " column, and headed " $M_x$ ." Then  $M_x = C_x + C_{x+1} + C_{x+2} + \text{etc.}$ , to the limit of the table, and the formula for the whole life single premium becomes  $A_x = \frac{M_x}{D_x}$ .

" $S_x$ " AND " $R_x$ " COLUMNS:— Besides the four columns above described, two others based on them are sometimes useful.

The " $S_x$ " column is formed by summing the values in the " $N_x$ " column in the same way that the " $N_x$ " column is formed by summing the values in the " $D_x$ " column. Thus, by the American Experience Table,  $S_{95} = N_{95} = D_{95}$ ;  $S_{94} = N_{94} + N_{95} = (D_{94} + D_{95}) + D_{95} = D_{94} + 2D_{95}$ ;  $S_{93} = N_{93} + N_{94} + N_{95} = (D_{93} + D_{94} + D_{95}) + (D_{94} + D_{95}) + D_{95} = D_{93} + 2D_{94} + 3D_{95}$ ; and so forth. There is, however, so little practical use for the " $S_x$ " column that it seemed best not to include values for it in the tables.

The " $R_x$ " column is formed by summing the values in the " $M_x$ " column in the same way as indicated for the " $S_x$ " column. Thus by the American Experience Table:  $R_{95} = M_{95} = C_{95}$ ;  $R_{94} = M_{94} + M_{95} = (C_{94} + C_{95}) + C_{95} = C_{94} + 2C_{95}$ ;  $R_{93} = M_{93} + M_{94} + M_{95} = (C_{93} + C_{94} + C_{95}) + (C_{94} + C_{95}) + C_{95} = C_{93} + 2C_{94} + 3C_{95}$ ; and so on.

It is very important to clearly understand the first four columns above indicated, the  $D_x$ ,  $N_x$ ,  $C_x$ , and  $M_x$  columns, for they are made the basis of all ordinary life insurance formulas and calculations. No attempt, however, should be made to give the values in the Commutation Tables any "meaning," for they constitute simply a mathematical device for the saving of labor.

NOTE.—In the foregoing chapter, the method shown for computing the  $N$  column is that which agrees with the more common practice in America. The British custom is to make  $N_x = D_{x+1} + D_{x+2} + \text{etc.}$ , omitting the term  $D_x$ . This does not raise a serious difficulty, because it will be seen that the British  $N_x$  is the same as the American  $N_{x+1}$ ; and similarly the American  $N_x$  is the same as the British  $N_{x-1}$ . It is only necessary to know which  $N$  is intended, and this can be found by glancing at the  $N$  for the highest age of the table. In the American Experience Table, by American practice,  $N_{95} = D_{95}$ , and has some value. By British practice  $N_{95} = D_{96}$ , which of course is zero.

## CHAPTER VII.

## NET PREMIUM FORMULAS STATED IN COMMUTATION SYMBOLS.

SINGLE PREMIUM FOR WHOLE LIFE INSURANCE:—As this formula was used to show the application of the Commutation Tables it is here given, without explanation, for the sake of comparison:

$$\begin{aligned}
 A_x &= \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \text{etc., to table limit}}{l_x} \\
 &= \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + \text{etc., to table limit}}{v^x l_x} \\
 &= \frac{C_x + C_{x+1} + C_{x+2} + \text{etc., to table limit}}{D_x} \\
 &= \frac{M_x}{D_x}
 \end{aligned}$$

SINGLE PREMIUM FOR TERM INSURANCE:—Here the insurance is to continue for  $n$  years only, and we therefore have only  $n$  terms in the numerator. This formula is

$$\begin{aligned}
 &\frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \dots + v^{n-1}d_{x+n-2} + v^nd_{x+n-1}}{l_x} \\
 \text{or } &\frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + \dots + v^{x+n-1}d_{x+n-2} + v^{x+n}d_{x+n-1}}{v^x l_x},
 \end{aligned}$$

the final term in the numerator being what it is because, if the insurance is to continue only  $n$  years, the last deaths considered will be those of persons who were a year previous  $n-1$  years older than at the outset, and because the payments on account of those deaths will fall due just at the end of  $n$  years from the beginning of the contract.

In Commutation symbols we then have

$$\frac{C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-2} + C_{x+n-1}}{D_x}$$

Now  $M_x = C_x + C_{x+1} + C_{x+2} + C_{x+3} + \text{etc., to table limit}$ ,  
 and  $M_{x+n} = C_{x+n} + C_{x+n+1} + C_{x+n+2} + \text{etc., to table limit}$ ,  
 $C_{x+n-1}$  being the first value, counting from the table limit, which is contained in  $M_x$  but not in  $M_{x+n}$ .

Therefore,  $C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-2} + C_{x+n-1} = M_x - M_{x+n}$ , and we have as our single premium for term insurance for  $n$  years:—

$$\frac{M_x - M_{x+n}}{D_x}.$$

**SINGLE PREMIUM FOR PURE ENDOWMENT:**—Here a sum is payable only in case the insured is living at the expiration of a term of years, and the formula is  $\frac{v^n l_{x+n}}{l_x}$ . The  $l_{x+n}$  persons each get \$1  $n$  years hence and the sum thus represented must, therefore, be discounted for  $n$  years and then collected in equal parts from each of the  $l_x$  persons living now.

Multiplying numerator and denominator by  $v^x$  we have the equivalent fraction  $\frac{v^{x+n} l_{x+n}}{v^x l_x}$ ; which in Commutation symbols is  $\frac{D_{x+n}}{D_x}$ .

**SINGLE PREMIUM FOR "ENDOWMENT INSURANCE" OR "ENDOWMENT":**—This form of policy is merely a combination of the last two previous, and its single premium is found by adding the single premiums for those contracts; thus

$$\frac{M_x - M_{x+n}}{D_x} + \frac{D_{x+n}}{D_x} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}.$$

We have covered the single premiums for all the usual forms of policy, and now take up the subject of annual premiums for those policies.

**ANNUAL PREMIUMS:**—Our rule, previously outlined, for finding an annual premium, is:—"Divide the single premium for the insurance by the present value at the same age of a series of annual payments of \$1 (the first immediate) for life or a designated term of life. The result will be the annual premium during life, or during such term, as the case may be."

**ANNUAL PREMIUM FOR WHOLE LIFE INSURANCE:**—The Single Premium is  $\frac{M_x}{D_x}$  and the P. V. of the life series of \$1 annual payments, "annuity-due," is  $\frac{N_x}{D_x}$ . The Annual Premium is therefore

$$\frac{M_x}{D_x} \div \frac{N_x}{D_x} = \frac{M_x}{D_x} \times \frac{D_x}{N_x} = \frac{M_x}{N_x}.$$



ANNUAL PREMIUM LIMITED TO  $n$  YEARS FOR WHOLE LIFE INSURANCE:—The Single Premium is  $\frac{M_x}{D_x}$ , and the P. V. of the  $n$ -year series of \$1 annual payments is  $\frac{N_x - N_{x+n}}{D_x}$ . The Annual Premium is therefore  $\frac{M_x}{D_x} \div \frac{N_x - N_{x+n}}{D_x} = \frac{M_x}{N_x - N_{x+n}}$ .

ANNUAL PREMIUM FOR TERM INSURANCE FOR  $n$  YEARS:—The Single Premium is  $\frac{M_x - M_{x+n}}{D_x}$ , and the P. V. of the  $n$ -year series of \$1 annual payments is  $\frac{N_x - N_{x+n}}{D_x}$ . The Annual Premium is therefore

$$\frac{M_x - M_{x+n}}{D_x} \div \frac{N_x - N_{x+n}}{D_x} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}.$$

ANNUAL PREMIUM FOR  $n$ -YEAR ENDOWMENT:—The Single Premium is  $\frac{M_x - M_{x+n} + D_{x+n}}{D_x}$ , and the P. V. of the  $n$ -year series of \$1 annual payments is  $\frac{N_x - N_{x+n}}{D_x}$ . The Annual Premium is therefore  $\frac{M_x - M_{x+n} + D_{x+n}}{D_x} \div \frac{N_x - N_{x+n}}{D_x} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}$ .

\*RETURN PREMIUM INSURANCE:—This is a form of policy which provides, in addition to the promises contained in either a life or endowment form, that, if death occur during a certain limited term, all premiums paid up to the time of death shall be repaid. In other words, if the policy were for \$1,000, the gross premium paid yearly were \$30, and the time thus limited were ten years, then if the insured died the first year, there would be payable to his representatives \$1,000 + \$30 = \$1,030, as he would have paid one premium. In the second year the sum payable would be \$1,000 + \$60 = \$1,060, as two premiums would have been paid. In the 10th year the sum would be \$1,300, because 10 premiums of \$30 each would then have been paid. If, however, he should die in any year of the policy after the 10th, the sum payable would be simply the \$1,000 without

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\*As the formulas for Return Premium Insurance are rather complex, and these plans of insurance are seldom met with, it may be well to omit their study until the other formulas in this and the following chapters are thoroughly understood.

addition of any premiums. It will be seen that the calculation of the premium for this contract involves a "gross" or "office" premium—i. e., the net premium with the addition made to it to cover expenses, etc.—for that is the premium which is to be returned. Now if we call the net annual premium  $P_x$  and consider the loading or margin as a percentage of that net premium, we can call the gross premium  $gP_x$ , where  $g$  is equal to 1.20 or 1.25 or 1.30, etc.

Let us first take the case of an  $n$ -payment whole life insurance with return of all premiums paid during  $n$  years. The formula for the annual premium is as follows:—

$$P_x = \frac{M_x}{N_x - N_{x+n}} + \frac{gP_x C_x + 2gP_x C_{x+1} + 3gP_x C_{x+2} + \dots + ngP_x C_{x+n-1}}{N_x - N_{x+n}}$$

The first fractional term covers simple  $n$ -payment whole life insurance. The second term covers the return of premiums, and

may be simplified to,  $gP_x \frac{C_x + 2C_{x+1} + 3C_{x+2} + \dots + nC_{x+n-1}}{N_x - N_{x+n}}$ , or

$$gP_x \frac{(C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-1}) + (C_{x+1} + C_{x+2} + \dots + C_{x+n-1}) + (C_{x+2} + \dots + C_{x+n-1}) + (C_{x+3} + \dots + C_{x+n-1}) + \text{etc.} + C_{x+n-1}}{N_x - N_{x+n}}$$

$$\text{or } gP_x \frac{(M_x - M_{x+n}) + (M_{x+1} - M_{x+n}) + \text{etc.} + (M_{x+n-1} - M_{x+n})}{N_x - N_{x+n}}$$

$$\text{or } gP_x \frac{(M_x + M_{x+1} + M_{x+2} + M_{x+3} + \dots + M_{x+n-1}) - nM_{x+n}}{N_x - N_{x+n}}.$$

Now, if we sum all the values of  $M$  from the highest age of the table down to and including  $M_x$  we have the Commutation Table value  $R_x$ . If, however, we stop the summation with  $M_{x+n}$  we have  $R_{x+n}$ . Evidently then  $(M_x + M_{x+1} + \dots + M_{x+n-1}) = R_x - R_{x+n}$ , and our complete formula for  $P_x$  then reads:

$$P_x = \frac{M_x}{N_x - N_{x+n}} + gP_x \frac{R_x - R_{x+n} - nM_{x+n}}{N_x - N_{x+n}}$$

or by algebraic transformation,

$$P_x (N_x - N_{x+n}) = M_x + gP_x (R_x - R_{x+n} - nM_{x+n})$$

$$\text{or } P_x [(N_x - N_{x+n}) - g(R_x - R_{x+n} - nM_{x+n})] = M_x$$

or finally

$$P_x = \frac{M_x}{N_x - N_{x+n} - g(R_x - R_{x+n} - nM_{x+n})}.$$

The above somewhat complicated formula is for the net annual premium for  $n$ -payment whole life insurance with return of premiums paid during  $n$  years.

ENDOWMENT WITH RETURN OF PREMIUMS:—If we consider  $P_x$  as the net annual premium and  $gP_x$  as the corresponding gross premium, the formula for an  $n$ -year endowment with return of premiums paid during  $n$  years is as follows:

$$P_x = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n} - g(R_x - R_{x+n} - nM_{x+n})}$$
, which differs only in the numerator from the preceding formula.

# CHAPTER VIII.

## FORMULAS FOR NET VALUATION.

WE will now give in algebraic form some expressions for the amount of reserve at various periods. Our general rule, given at the close of Chapter IV, is as follows:—

“The net reserve, or ‘net value,’ at the end of any year is to be found by deducting from the net single premium for the insurance, at the attained age of the insured, the then present value of the net premiums yet to be received. The balance will be the net reserve.”

Suppose a whole life insurance is issued at age  $x$  with annual premiums. The annual premium would be  $\frac{M_x}{N_x}$ . At the end of

$m$  years the insured would be  $x + m$  years of age, and the single premium for whole life insurance at age  $x + m$  is  $A_{x+m}$ . The present value at age  $x + m$  of an annuity-due of \$1 would be  $\frac{N_{x+m}}{D_{x+m}}$ , or  $(1 + a_{x+m})$ , and the present value at age  $x + m$  of

a life series of payments of  $\frac{M_x}{N_x}$  would therefore be  $\frac{M_x}{N_x} \times \frac{N_{x+m}}{D_{x+m}}$ .

Then we have the formula for the reserve on a whole life policy as follows:

$${}_mV_x = \frac{M_{x+m}}{D_{x+m}} - \frac{M_x}{N_x} \cdot \frac{N_{x+m}}{D_{x+m}}, \text{ or in shorter form } {}_mV_x = A_{x+m} -$$

$$\frac{M_x}{N_x}(1 + a_{x+m}), \text{ where } {}_mV_x \text{ means the reserve at the end of } m$$

years on a policy issued at age  $x$ .

This formula is subject to considerable variation. For instance:—as

$$\frac{M_{x+m}}{N_{x+m}} = \frac{M_{x+m}}{D_{x+m}} \div \frac{N_{x+m}}{D_{x+m}},$$

$$\text{and } \frac{M_{x+m}}{D_{x+m}} = A_{x+m} = \frac{M_{x+m}}{N_{x+m}} \times \frac{N_{x+m}}{D_{x+m}} = \frac{M_{x+m}}{N_{x+m}}(1 + a_{x+m})$$

$$\text{therefore } {}_mV_x = \frac{M_{x+m}}{N_{x+m}}(1 + a_{x+m}) - \frac{M_x}{N_x}(1 + a_{x+m})$$

$$= \left( \frac{M_{x+m}}{N_{x+m}} - \frac{M_x}{N_x} \right) (1 + a_{x+m}).$$



Therefore if we have tables of single premiums, annual premiums and annuities—due we may find the reserve for a whole life insurance policy at any assumed age and at the end of any designated number of years from the issue of the contract.

To find the reserve at the end of 10 years for an annual premium whole life policy for \$1 issued at age 20:—

Using the first formula,  ${}_mV_x = A_{x+m} - \frac{M_x}{N_x}(1 + a_{x+m})$ ; in which if  $x = 20$  and  $m = 10$  we have:  ${}_{10}V_{20} = A_{30} - \frac{M_{20}}{N_{20}}(1 + a_{30})$ .

If we use the second formula we have:

$${}_{10}V_{20} = \left( \frac{M_{30}}{N_{30}} - \frac{M_{20}}{N_{20}} \right) (1 + a_{30})$$

If premiums for the whole life insurance are to be limited to  $n$  years the expression takes the form, as explained below:

$${}_mV_x = \frac{M_{x+m}}{D_{x+m}} - \frac{M_x}{N_x - N_{x+n}} \cdot \frac{N_{x+m} - N_{x+n}}{D_{x+m}},$$

in which  $\frac{M_{x+m}}{D_{x+m}} = A_{x+m}$  is the single premium at the attained age  $x + m$ ;  $\frac{M_x}{N_x - N_{x+n}}$  is the annual premium charged, for  $n$  years altogether; and  $\frac{N_{x+m} - N_{x+n}}{D_{x+m}}$  is the P. V. of an annuity-

due beginning at the attained age and lasting only so long as the premium-paying period of the policy.

For an  $n$ -year endowment the formula for the reserve at the end of  $m$  years is as follows:

$${}_mV_x = \frac{M_{x+m} - M_{x+n} + D_{x+n}}{D_{x+m}} - \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \cdot \frac{N_{x+m} - N_{x+n}}{D_{x+m}},$$

in which the first term is the single premium at the attained age  $x + m$  for an endowment insurance to terminate with the attainment of age  $x + n$ , as provided in the policy under consideration; the second term is the annual premium charged for  $n$  years in all; and the third is the P. V. of an annuity-due, beginning at the attained age of valuation,  $x + m$ , and continuing for what remains of the premium-paying period of the policy.

The reserve for any form of policy at any assumed period may be found by computing the single premium at the attained age of the insured for the same sort of benefit, to terminate at the same

time as the original benefit, and deducting from this new single premium the then present value of the future premiums on the original policy. When no more premiums are payable on a policy, its reserve is then simply the single premium at the attained age.

ACCUMULATION FORMULA:—Another very useful formula for obtaining reserves will now be shown.

At the beginning of a policy's existence, or as soon as it goes into force, the reserve on the policy will be the amount of the net premium which has just been received by the company. Thus if a policy for \$1 is issued at age  $x$  with a net premium of  $P_x$  this "Initial Reserve," as it is called, would be  $P_x$ . During the course of the first year interest  $i$  would be earned, so that at the end of the year,  $P_x$  would have become  $P_x(1+i)$ , because of interest. If  $l_x$  persons each paid  $P_x$ ,  $l_x P_x$  would be received, and this would become  $l_x P_x (1+i)$  at the close of the year. If this fund were at that time apportioned equally among the surviving  $l_{x+1}$  persons, the contingent share of each would be  $\frac{l_x P_x (1+i)}{l_{x+1}}$ . The deaths

in this first year, however, would be  $d_x$  out of the  $l_x$  who entered, and  $\$d_x$  would therefore be payable at the close of the year out of any funds received from the  $l_x$  persons. If this liability were apportioned equally among the  $l_{x+1}$  survivors, the share of each would be  $\frac{d_x}{l_{x+1}}$ . Then if we deduct from  $\frac{l_x}{l_{x+1}}P_x(1+i)$ , which is each survivor's gross contingent share of the funds received, the sum of  $\frac{d_x}{l_{x+1}}$ , which is each survivor's proportion of the amount to be paid in death claims, we have as a balance each survivor's contingent share in the fund remaining. This share is the reserve on his policy at the close of the first year.

Then in algebraic form we have

$${}_1V_x = \frac{l_x}{l_{x+1}}P_x(1+i) - \frac{d_x}{l_{x+1}},$$

It is not necessary that there be exactly  $l_x$  persons entering, or exactly  $d_x$  deaths in the year, for as will be seen in the formula, every value is merely proportioned to the number living and number dying according to the mortality table. If the insurances were for \$1,000 instead of \$1, both the premium and the amount of death claims would be just 1,000 times as great, and the reserve would therefore be \$1,000  ${}_1V_x$ .

If we note that, as  $\frac{1}{1+i} = v$ , then  $1+i = \frac{1}{v}$ , we can substitute this value in the first term, making it  $\frac{l_x}{vl_{x+1}} P_x$ . Then if we introduce in numerator and denominator the factor  $v^x$ , we have  $\frac{v^x l_x}{v^{x+1} l_{x+1}} P_x = \frac{D_x}{D_{x+1}} P_x$ . In the same way if we multiply both numerator and denominator of the second term by  $v^{x+1}$  we have  $\frac{v^{x+1} d_x}{v^{x+1} l_{x+1}} = \frac{C_x}{D_{x+1}}$ .

The full formula is then:  ${}_1V_x = \frac{D_x}{D_{x+1}} P_x - \frac{C_x}{D_{x+1}}$ , and the values of the fractions can readily be found from the commutation tables.

We will now find the reserve at the end of the second year, or  ${}_2V_x$ . At the beginning of this year, the company will have on hand for each person insured,  ${}_1V_x$ , and  $P_x$ , the premium just received. This  $({}_1V_x + P_x)$  will become, at interest  $i$ ,  $({}_1V_x + P_x)(1+i)$ , at the close of the year. If we then apportion this fund as before in the proportion shown by the mortality table, of  $l_{x+1}$  persons entering this year of life and  $l_{x+2}$  surviving it, the gross share of each survivor would be  $\frac{l_{x+1}}{l_{x+2}} ({}_1V_x + P_x)(1+i)$ . From

this should be deducted the individual survivor's share of death claims in the proportion shown by the mortality table, which is  $\frac{d_{x+1}}{l_{x+2}}$ , and the remainder is the reserve at the close of the policy's

second year. Then we have  ${}_2V_x = \frac{l_{x+1}}{l_{x+2}} ({}_1V_x + P_x)(1+i) - \frac{d_{x+1}}{l_{x+2}}$ .

The first part of this expression may be made to read  $\frac{v^{x+1} l_{x+1}}{v^{x+2} l_{x+2}} ({}_1V_x + P_x)$ , and the second part  $\frac{v^{x+2} d_{x+1}}{v^{x+2} l_{x+2}}$ .

Therefore  ${}_2V_x = \frac{D_{x+1}}{D_{x+2}} ({}_1V_x + P_x) - \frac{C_{x+1}}{D_{x+2}}$ .

In the same way:  ${}_3V_x = \frac{D_{x+2}}{D_{x+3}} ({}_2V_x + P_x) - \frac{C_{x+2}}{D_{x+3}}$ .

It will be seen that the fractions  $\frac{D_x}{D_{x+1}}, \frac{D_{x+1}}{D_{x+2}}, \frac{D_{x+2}}{D_{x+3}}$ , etc., are symmetrical in form, the denominator in each case being the  $D$  for an age one year older than for the numerator. It is also clear

that they are independent of the form of policy, amount of insurance, or premium. Therefore their numerical values can conveniently be worked out for each age and tabulated. The symbol used for this fraction is  $u_x$ . In other words,

$$u_x = \frac{D_x}{D_{x+1}}, u_{x+1} = \frac{D_{x+1}}{D_{x+2}}, \text{ etc., for any age we assign to } x.$$

Similarly the fractions  $\frac{C_x}{D_{x+1}}, \frac{C_{x+1}}{D_{x+2}}, \frac{C_{x+2}}{D_{x+3}}$ , are symmetrical and independent of the policy form and premium, and their values may be tabulated for each age. The symbol for this fraction is  $k_x$ .

Numerical values for  $u_x$  and  $k_x$  are tabulated at the end of this book. It is to be noted that, as  $k_x$  contains no interest element, it remains the same for all rates of interest taken with the same mortality table.

Making substitution of these new symbols in the formulas, we have:—

$${}_1V_x = P_x u_x - k_x$$

$${}_2V_x = ({}_1V_x + P_x) u_{x+1} - k_{x+1}$$

$${}_3V_x = ({}_2V_x + P_x) u_{x+2} - k_{x+2}; \text{ or in general}$$

$${}_mV_x = ({}_{m-1}V_x + P_x) u_{x+m-1} - k_{x+m-1},$$

$${}_{m+1}V_x = ({}_mV_x + P_x) u_{x+m} - k_{x+m}, \text{ and}$$

$$1,000 {}_mV_x = (1,000 {}_{m-1}V_x + 1,000 P_x) u_{x+m-1} - 1,000 k_{x+m-1},$$

$$1,000 {}_{m+1}V_x = (1,000 {}_mV_x + 1,000 P_x) u_{x+m} - 1,000 k_{x+m}, \text{ etc.,}$$

which is a very simple formula.

It should be observed that this formula is absolutely independent of the form of policy so long as the amount of insurance does not vary. If the amount payable in any year in case of death in that year varies from the unit of \$1, \$1,000, \$10,000, etc., on which the premium is based, the factor of  $k_x$  must be changed to agree with the true conditions. Thus, if the amount payable because of death in a particular year is \$1,050 instead of the \$1,000 on which the premium is based, the multiple of  $k_x$  should be \$1,050 and not \$1,000. This point comes up in connection with return premium policies.

It may have occurred to the reader that the accumulation formula follows out for the individual policy the same sort of account that was shown in Chapter IV for the group according to the mortality table. This is true, and if the fund on hand at the close of each year be divided by the number of survivors of that year, the result as shown in the last column of that table will be the same as would be found by using this formula.



## CHAPTER IX.

## ANNUITIES.

BESIDES the life insurance contracts outlined in the previous chapters many companies offer annuity policies. An annuity contract is one providing for a series of periodical payments by the company during the continued life of the annuitant or annuitants, or during a term of life.

A Single Premium Whole Life Annuity is such a contract paid for in one lump sum, to continue during the life of the annuitant.

If we assume that  $l_x$  persons aged  $x$  made similar contracts by which the company was bound to pay \$1 at the end of each year of their after-life,  $l_{x+1}$  persons would survive one year, and the present value of this sum payable a year hence would be  $v l_{x+1}$ ;  $l_{x+2}$  persons would still be living at the close of the second year and the company would have to pay  $l_{x+2}$  dollars, the present value of this payment being  $v^2 l_{x+2}$ . The sum to be paid at the close of the third year would be  $l_{x+3}$ , and its present value would be  $v^3 l_{x+3}$ . Similar terms,  $v^4 l_{x+4}$ ,  $v^5 l_{x+5}$ , etc., show the present values of the sums payable at the close of further years of life. As each of the  $l_x$  persons entering has an equal chance of living to the highest age in the mortality table, each bears an equal share of the aggregate cost of the annuities, and this result is obtained by dividing the sum of this series of present values by  $l_x$ , the number entering into the contract.

The formula for the Single Premium—or Present Value—of this policy, denoted by  $a_x$ , for an annuity of \$1 yearly, then is

$$a_x = \frac{v l_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + v^4 l_{x+4} + \text{etc. to table limit}}{l_x}.$$

Multiplying numerator and denominator by  $v^x$  we have:

$$a_x = \frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + \text{etc. to table limit}}{v^x l_x};$$

or, in Commutation symbols:

$$a_x = \frac{D_{x+1} + D_{x+2} + D_{x+3} + \text{etc. to table limit}}{D_x};$$

$$\text{or } a_x = \frac{N_{x+1}}{D_x}, \text{ which, as } N_{x+1} = N_x - D_x, \text{ differs only by } \frac{D_x}{D_x},$$

or 1, from  $\frac{N_x}{D_x}$ , or  $1 + a_x$ , which is the whole life annuity-due, used previously in calculating annual premiums.

Therefore, when values of an annuity-due or "annuity, first payment immediate" are stated, we may find the simple "annuity" by deducting from the value as stated, the amount of the payment to be made immediately. When annuities are tabulated so as to give values up to the highest age of the mortality table, it is always possible to determine whether or not an annuity-due is intended to be stated, for on the American Table the value of an annuity-due of \$1 at the highest age of the living, 95, would be \$1; there being according to assumptions no possibility of a further payment, so that the value at that age of an annuity with its first payment a year hence would be zero. In the Actuaries Table the above rule would apply at age 99.

**TEMPORARY ANNUITY POLICY:**—This contract is similar to the life annuity, but payments for only a term of years are included. For an annuity of \$1 yearly, to continue for  $n$  years of life at age  $x$  the formula is:

$$\begin{aligned} & \frac{vl_{x+1} + v^2l_{x+2} + \text{etc.} + v^nl_{x+n}}{v^{x+1}l_{x+1} + v^{x+2}l_{x+2} + \text{etc.} + v^{x+n}l_{x+n}} \\ \text{or } & \frac{D_{x+1} + D_{x+2} + \text{etc.} + D_{x+n}}{D_x} \\ \text{or } & \frac{N_{x+1} - N_{x+n+1}}{D_x} \end{aligned}$$

The same direct relation to a temporary annuity-due,—such as we have had occasion to use in connection with premiums for limited terms,—does not here exist, as in the case of the whole life annuity.

The formula for a "temporary" or "term" annuity-due for an  $n$ -year term is  $\frac{N_x - N_{x+n}}{D_x}$ , which includes the same number of payments— $n$ —but, as the first is immediate and is represented by  $\frac{D_x}{D_x}$ , the last contingent payment, represented in the previous fraction by  $\frac{D_{x+n}}{D_x}$ , is dropped.

**DEFERRED ANNUITIES:**—Under this form of policy the first payment of annuity is not to take place until a

certain fixed time has elapsed. If the time which is to pass before the first payment is taken as  $n$  years, and the payments are to continue thereafter for life, the formula for the present value is  $\frac{v^n l_{x+n} + v^{n+1} l_{x+n+1} + v^{n+2} l_{x+n+2} + \text{to table limit}}{l_x}$ ,

$$\text{or } \frac{v^{x+n} l_{x+n} + v^{x+n+1} l_{x+n+1} + v^{x+n+2} l_{x+n+2} + \text{to table limit}}{v^x l_x},$$

$$\text{or } \frac{D_{x+n} + D_{x+n+1} + D_{x+n+2} + \text{etc., to table limit}}{D_x},$$

$$\text{or } \frac{N_{x+n}}{D_x}.$$

If then we have the present value at age  $x$  of a life annuity-due and also of an annuity-due limited to  $n$  payments, we may find the above deferred annuity by deducting from the P. V. of the life annuity-due,  $\frac{N_x}{D_x}$ , the P. V. of the term annuity-due,  $\frac{N_x - N_{x+n}}{D_x}$ , leaving  $\frac{N_{x+n}}{D_x}$ .

Deferred annuities to be paid for by single premiums are often found in combination with what are called "annuity-certain" contracts. Under such an agreement the company is bound to make a certain limited number of payments irrespective of the life or death of the named beneficiary, and any further payments are contingent on the life of that person at the times such payments would fall due. Such an arrangement is frequently provided for in policies as a "Continuous Instalment Option," as an alternative to the payment, in one sum, of an amount due under a death claim. In this case the amount of annuity purchased by this sum depends on the age of the person named as beneficiary when the policy matures by the insured's death.

Deferred annuities may be paid for by annual premiums which are to continue up to a year previous to the first payment under the annuity. The annual premium for this contract would be

$$\frac{N_{x+n}}{D_x} \div \frac{N_x - N_{x+n}}{D_x} = \frac{N_{x+n}}{N_x - N_{x+n}}.$$

Besides the annuities outlined above there are many special forms involving two or more lives, but the limits of this elementary book do not permit of more than a definition of their general character. It is sufficient to note that they are based on the same general principles as those applying to annuities on single lives, except that contingencies regarding more than one life are taken into account.

A "Joint Life Annuity" provides for payments only so long as *all* the persons named survive.

An "Annuity Jointly and to the Last Survivor" provides for payments as long as any one of the persons named is living.

Under a "Survivorship Annuity" payments are to be made only during the years that a certain designated person, or group of persons, outlives another designated person or group of persons. Otherwise no payments are made. Such a contract may be purchased by a man in favor of his wife at small cost by annual premiums. Supposedly during his life he can support her, and if he dies first this form of contract provides her a life income from the time of his death. The first and third of these contracts may be combined in one.

It has been found by actual experience that mortality tables, such as those given in this book, on the basis of insured lives, do not state correctly the probabilities of living and dying among persons on whose lives annuities have been issued. This is particularly the case where the payments under the annuity are to begin within a short time from the present. "Annuitants," as they are called, have much greater longevity than insured persons, so that, if premiums for annuity policies were based on the mortality tables derived from experience with insured lives, they would suppose a less number of persons living in future years than actually would be living, and would therefore be in deficit. Evidently no one takes out an annuity policy unless one feels that one's health and vitality are such as to give grounds for an expectation of long life. The longevity of women annuitants is so much greater than that of men that they are charged higher premiums, though the longevity of *insured* women is less than that of men. It may be that the annuities themselves furnish those comforts and luxuries which conduce to long life.

We take up, in closing, one other form of annuity policy—that of a Life Annuity Payable Oftener than Once a Year. As our mortality table gives us no definite data as to the probabilities of living fractions of a year, we make use of an approximation which is sufficiently accurate for practical purposes.

Let us first consider an annuity of 1 payable in  $m$  instalments so that the instalments falling in each year shall be  $\frac{1}{m}$  each and fall  $\frac{1}{m}$  of a year apart, the first coming  $\frac{1}{m}$  of a year from now, the second  $\frac{2}{m}$  of a year hence, the third  $\frac{3}{m}$  of a year hence, and the



last instalment of the first yearly payment of 1 at the close of a year from now. In the following years corresponding payments of instalments would fall due.

We know already that the P. V. of an annuity-due of 1 yearly is  $1 + a_x$ , or  $\frac{N_x}{D_x}$ , and that the P. V. of an annuity of 1, having its first payment a year hence, is  $a_x$ . Then, by proportion, the P. V. of an annuity having payments due  $\frac{1}{m}$  of a year hence and annually from that date would have a P. V., intermediate between  $1 + a_x$  and  $a_x$ , of  $1 + a_x - \frac{1}{m}$ , or  $\frac{m-1}{m} + a_x$ . If payments were to begin  $\frac{2}{m}$  of a year hence the value would be  $1 + a_x - \frac{2}{m}$ , or  $\frac{m-2}{m} + a_x$ , and so on. If the payments under each of these latter annuities were to be  $\frac{1}{m}$  instead of 1, their present values would be  $\frac{1}{m} \left( \frac{m-1}{m} + a_x \right)$ ,  $\frac{1}{m} \left( \frac{m-2}{m} + a_x \right)$ , and so on.

We would then have under the original proposition the sum of a series of  $m$  present values as follows:

$$\frac{1}{m} \left( \frac{m-1}{m} + a_x \right) + \frac{1}{m} \left( \frac{m-2}{m} + a_x \right) + \frac{1}{m} \left( \frac{m-3}{m} + a_x \right) + \text{etc.,} + \frac{1}{m} \left( \frac{m-m}{m} + a_x \right),$$

$$\text{or } \frac{1}{m} \left[ \left( \frac{m-1}{m} + a_x \right) + \left( \frac{m-2}{m} + a_x \right) + \left( \frac{m-3}{m} + a_x \right) + \text{etc.,} + a_x \right].$$

The latter consists of an arithmetical series multiplied by  $\frac{1}{m}$ , and the rule for finding the sum of such a series is:—"Multiply the sum of the first and last terms by the number of terms and divide the product by two." Here the first term (within brackets) is  $\left( \frac{m-1}{m} + a_x \right)$  and the last is  $a_x$ . The number of terms is  $m$ .

Then we have, as the sum of the series, within brackets, the expression  $\frac{m \left[ \left( \frac{m-1}{m} + a_x \right) + a_x \right]}{2}$ , and  $\frac{1}{m}$  of that is  $\frac{\left( \frac{m-1}{m} + a_x \right) + a_x}{2}$ ,

or  $\frac{\frac{m-1}{m} + 2a_x}{2}$ , or  $\frac{m-1}{2m} + a_x$ , or  $\frac{m-1}{2m} + \frac{N_{x+1}}{D_x}$ , which is, therefore, the P. V., of an annuity of 1 payable in  $m$  instalments of  $\frac{1}{m}$  each, falling due  $\frac{1}{m}$  of a year hence and at intervals of  $\frac{1}{m}$  of a year thereafter during the life of a person aged  $x$ .

For an annuity of 1 payable in semi-annual instalments of  $\frac{1}{2}$  each, the formula would be  $\frac{2-1}{2 \times 2} + a_x$ , or  $\frac{1}{4} + a_x$ .

For an annuity payable in quarterly instalments of  $\frac{1}{4}$  each, the formula would be  $\frac{4-1}{2 \times 4} + a_x$ , or  $\frac{3}{8} + a_x$ .

## CHAPTER X.

## REVIEW OF FORMULAS BY ACTUAL CALCULATIONS.

Now that the various formulas have been enunciated it may be well to show their practical application by working out actual values from the Commutation Columns. We first take up formulas for certain single and annual premiums.

Single Premium for Whole Life Insurance,  $\frac{M_x}{D_x} = A_x$ .

- (1) Let  $x = 20$ , and the basis be American 3 per cent.

$$\frac{M_{20}}{D_{20}} = \frac{16,974.076,5}{51,290.86} = \$0.330,94 \text{ per } \$1, \text{ and } \$330.94 \text{ per } \$1,000.$$

- (2) Let  $x = 36$ , and the basis be Actuaries 4 per cent.

$$\frac{M_{36}}{D_{36}} = \frac{6,940.968,52}{19,935.512,81} = \$0.348,17 \text{ per } \$1, \text{ and } \$348.17 \text{ per } \$1,000.$$

Annual Premium during Life for Whole Life Insurance,  $\frac{M_x}{N_x}$ .

- (3) Let  $x = 20$ , and the basis be American 3 per cent.

$$\frac{M_{20}}{N_{20}} = \frac{16,974.076,5}{1,178,209.61} = \$0.014,41 \text{ per } \$1, \text{ and } \$14.41 \text{ per } \$1,000.$$

- (4) Let  $x = 40$ , and the basis be Actuaries 4 per cent.

$$\frac{M_{40}}{N_{40}} = \frac{6,242.419,04}{263,643.618,99} = \$0.023,68 \text{ per } \$1, \text{ and } \$23.68 \text{ per } \$1,000.$$

The equivalent formula in terms of the single premium and annuity-due is  $\frac{A_x}{1+a_x}$ .

- (5) Let  $x = 30$ , and the basis be Actuaries 4 per cent.

$$\frac{A_{30}}{1+a_{30}} = \frac{0.306,17}{18.040} = \$0.016,97 \text{ per } \$1, \text{ and } \$16.97 \text{ per } \$1,000.$$

Annual Premium Payable for  $n$  Years for Whole Life Insurance,

$$\frac{M_x}{N_x - N_{x+n}}.$$

(6) Let  $x = 30$ ,  $n = 20$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$N_{30} = 596,803.5$$

$$N_{50} = 181,663.3 \quad M_{30} = \frac{10,259.2}{415,140.2} = \$0.024,71 \text{ per } \$1 \text{ and}$$

$$N_{30} - N_{50} = \dots\dots\dots 415,140.2$$

\$24.71 per \$1,000.

(7) Let  $x = 35$ ,  $n = 15$ , and the basis be American 3 per cent.

$$N_{35} = 579,160.66$$

$$N_{50} = 243,156.01 \quad M_{35} = \frac{12,209.422,9}{336,004.65} = \$0.036,34 \text{ per } \$1,$$

$$N_{35} - N_{50} = \dots\dots\dots 336,004.65$$

and \$36.34 per \$1,000.

Single Premium for  $n$ -Year Term Insurance,  $\frac{M_x - M_{x+n}}{D_x}$ .

(8) Let  $x = 30$ ,  $n = 10$ , and the basis be American 3 per cent.

$$M_{30} = 13,574.817,7$$

$$M_{40} = 11,000.402,6$$

$$\frac{M_{30} - M_{40}}{D_{30}} = \frac{2,574.415,1}{35,200.56} = \$0.073,14 \text{ per } \$1, \text{ and } \$73.14 \text{ per}$$

\$1,000.

(9) Let  $x = 40$ ,  $n = 1$ , and the basis be American  $3\frac{1}{2}$  per cent.  
Then, as  $M_{40} - M_{41} = C_{40}$ , the formula is

$$\frac{C_{40}}{D_{40}} = \frac{186.69}{19,727.4} = \$0.009,46 \text{ per } \$1, \text{ and } \$9.46 \text{ per } \$1,000.$$

Annual Premium for  $n$ -Year Term Insurance,  $\frac{M_x - M_{x+n}}{N_x - N_{x+n}}$ .

(10) Let  $x = 40$ ,  $n = 20$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$M_{40} = 8,088.96$$

$$N_{40} = 344,167.3$$

$$M_{60} = 4,608.94$$

$$N_{60} = 81,106.4$$

$$M_{40} - M_{60} = \frac{3,480.02}{263,060.9} = \$0.013,23 \text{ per } \$1,$$

and \$13.23 per \$1,000.

Single Premium for  $n$ -Year Pure Endowment,  $\frac{D_{x+n}}{D_x}$ .



- (11) Let  $x = 45$ ,  $n = 15$ , and the basis be Actuaries 4 per cent.

$$\frac{D_{60} = 5,320.815,83}{D_{45} = 12,743.153,79} = \$0.417,54 \text{ per } \$1, \text{ and } \$417.54 \text{ per } \$1,000.$$

Single Premium for  $n$ -Year Endowment Insurance,

$$\frac{M_x - M_{x+n} + D_{x+n}}{D_x}.$$

- (12) Let  $x = 21$ ,  $n = 20$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$M_{21} = 12,916.5$$

$$M_{41} = 7,902.3$$

$$M_{21} - M_{41} = 5,014.2$$

$$D_{41} = 18,873.6$$

$$\frac{M_{21} - M_{41} + D_{41} = 23,887.8}{D_{21} = 44,630.8} = \$0.535,23 \text{ per } \$1, \text{ and } \$535.23 \text{ per } \$1,000.$$

- (13) Let  $x = 40$ ,  $n = 10$ , and the basis be American 3 per cent.

$$M_{40} = 11,000.402,6$$

$$M_{50} = 8,840.572,9$$

$$M_{40} - M_{50} = 2,159.829,7$$

$$D_{50} = 15,922.79$$

$$\frac{M_{40} - M_{50} + D_{50} = 18,082.62}{D_{40} = 23,943.93} = \$0.755,21 \text{ per } \$1, \text{ and } \$755.21 \text{ per } \$1,000.$$

Annual Premium for  $n$ -Year Endowment Insurance,

$$\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}.$$

- (14) Let  $x = 30$ ,  $n = 20$ , and the basis be American 3 per cent.

$$M_{30} = 13,574.817,7$$

$$M_{50} = 8,840.572,9$$

$$M_{30} - M_{50} = 4,734.24$$

$$N_{30} = 742,483.83$$

$$D_{50} = 15,922.79$$

$$N_{50} = 243,156.01 \quad M_{30} - M_{50} + D_{50} = 20,657.03$$

$$N_{30} - N_{50} = \dots\dots\dots 499,327.82$$

per \$1, and \$41.37 per \$1,000.

Single Premium for Whole Life Annuity with First Payment

Due One Year Hence,  $\frac{N_{x+1}}{D_x} = a_x$ .

(15) Let  $x = 20$ , and the basis be Actuaries 4 per cent.

$$\frac{N_{21} = 785,364.442,55}{D_{20} = 42,566.297,70} = \$18.450 \text{ for each } \$1 \text{ of annuity.}$$

Single Premium for  $n$ -Year Annuity with First Payment Due One Year Hence,  $\frac{N_{x+1} - N_{x+n+1}}{D_x}$ .

(16) Let  $x = 45$ ,  $n = 20$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$\begin{aligned} N_{46} &= 237,971.8 \\ N_{66} &= 43,343.08 \\ N_{46} - N_{66} &= \frac{194,628.7}{D_{45} = 15,773.6} = \$12.338,9 \text{ for each } \$1 \text{ of annuity.} \end{aligned}$$

Single Premium for Annuity with First Payment Deferred  $n$

Years,  $\frac{N_{x+n}}{D_x}$

(17) Let  $x = 40$ ,  $n = 20$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$\frac{N_{60} = 81,106.37}{D_{40} = 19,727.4} = \$4.111,4 \text{ for each } \$1 \text{ of annuity.}$$

Annual Premium for Annuity with Payment Deferred  $n$  Years,

$$\frac{N_{x+n}}{N_x - N_{x+n}}$$

(18) Let  $x = 25$ ,  $n = 25$ , and the basis be American  $3\frac{1}{2}$  per cent.

$$\begin{aligned} N_{25} &= 770,113.4 \\ N_{50} &= 181,663.3 & N_{50} &= 181,663.3 \\ N_{25} - N_{50} &= \dots\dots\dots 588,450.1 \end{aligned} = \$0.308,71 \text{ for each } \$1 \text{ of annuity.}$$

#### FORMULAS FOR NET RESERVES.

In all cases  $mV_x$  = Reserve at end of  $m$  years of a \$1 policy issued at  $x$ , and  $P_x$  = Proper net premium per \$1 of insurance for policy under consideration.

(The small number of decimal places used will be found to cause slight variations in some cases from values calculated with extreme accuracy.)

Reserve for Whole Life Policy with Annual Premiums for Life.

$${}_mV_x = A_{x+m} - P_x(1 + a_{x+m})$$

$$\text{or } {}_mV_x = (P_{x+m} - P_x)(1 + a_{x+m})$$

(19) Let  $x = 35$ ,  $m = 10$ , and the basis be American 3 per cent.

Then  $P_x = P_{35} = \$0.021,08$ ,  $1 + a_{x+m} = 1 + a_{45} = \$17.009,3$ , and

$$A_{x+m} = A_{45} = \$0.504,59$$

$$P_{35}(1 + a_{45}) = \$0.021,08 \times \$17.009,3 = .358,56$$

$$\text{and by the first formula, } {}_{10}V_{35} = \$0.146,03$$

$$1,000 {}_{10}V_{35} = \$146.03$$

Or,  $P_{x+m} = P_{45} = \$0.029,67$ , and  $P_{45} - P_{35} = \$0.029,67 - \$0.021,08 = \$0.008,59$ ;

and by the second formula  ${}_{10}V_{35} = (P_{45} - P_{35})(1 + a_{45}) = \$0.008,59 \times \$17.009,3 = \$0.146,11$ .  $1,000 {}_{10}V_{35} = \$146.11$

Reserve for Whole Life Policy with Annual Premiums Limited to  $n$  Years.

$${}_mV_x = A_{x+m} - P_x \cdot \frac{N_{x+m} - N_{x+n}}{D_{x+m}}$$

(20) Let  $x = 30$ ,  $n = 20$ ,  $m = 10$ , and the basis be American 3 per cent.

$$N_{x+m} = N_{40} = 444,394.39$$

$$A_{x+m} = A_{40} = \$0.459,42$$

$$N_{x+n} = N_{50} = 243,156.01$$

$$P_x = P_{30} = \$0.027,19$$

$$N_{40} - N_{50} = 201,238.38$$

$$D_{x+m} = D_{40} = \frac{201,238.38}{23,943.93} = 8.404,6 \times \$0.027,19 = .228,52$$

$${}_{10}V_{30} = \$0.230,90$$

$$1,000 {}_{10}V_{30} = \$230.90$$

Reserve for  $n$ -Year Endowment Insurance.

$${}_mV_x = \frac{M_{x+m} - M_{x+n} + D_{x+n}}{D_{x+m}} - P_x \cdot \frac{N_{x+m} - N_{x+n}}{D_{x+m}}$$

(21) Let  $x = 30$ ,  $n = 20$ ,  $m = 10$ , and the basis be American 3 per cent.

Then  $x + m = 40$ ,  $x + n = 50$ , and (from tables)  $P_x = P_{30} = \$0.041,37$ . From Example (13), we find the value of the first part of the formula to be  $\$0.755,21$ . From Example (20) we obtain the value of the 10-year annuity-due in the second part, which is  $\$8.404,6$ . Therefore,  ${}_{10}V_{30} = \$0.755,21 - (\$0.041,37 \times 8.404,6) = \$0.407,51$ , and  $1,000 {}_{10}V_{30} = \$407.51$ .

Accumulation Formula, applicable to all policies.

$${}_mV_x = ({}_{m-1}V_x + P_x) u_{x+m-1} - k_{x+m-1}$$

(22) Let  $x=35$ ,  $m=1, 2, 3$ , successively,  $P_x = P_{35} = \$0.021,08$  (the annual premium for Ordinary Whole Life), and the basis be American 3 per cent.

For  $m=1$ ,  ${}_{m-1}V_{35} = 0$ ,  $u_{35+m-1} = u_{35} = 1.039,298$

$$\$0.021,08 \times 1.039,298 = \$0.021,91$$

$$k_{35+m-1} = k_{35} = \underline{\underline{.009,03}}$$

$${}_1V_{35} = \$0.012,88, \quad 1,000 \, {}_1V_{35} = \$12.88.$$

$$P_{35} = \underline{\underline{.021,08}}$$

For  $m=2$ ,  $({}_{m-1}V_{35} + P_{35}) = \$0.033,96$ , and  $u_{35+m-1} = u_{36} = 1.039,447$

$$\$0.033,96 \times 1.039,447 = \$0.035,30$$

$$k_{35+m-1} = k_{36} = \underline{\underline{.009,17}}$$

$${}_2V_{35} = \$0.026,13 \quad 1,000 \, {}_2V_{35} = \$26.13.$$

$$P_{35} = \underline{\underline{.021,08}}$$

For  $m=3$ ,  $({}_{m-1}V_{35} + P_{35}) = \$0.047,21$ , and  $u_{35+m-1} = u_{37} = 1.039,600$

$$\$0.047,21 \times 1.039,600 = \$0.049,08$$

$$k_{35+m-1} = k_{37} = \underline{\underline{.009,32}}$$

$${}_3V_{35} = \$0.039,76 \quad 1,000 \, {}_3V_{35} = \$39.76.$$

The above examples are given merely by way of illustration. A student should construct formulas on other conditions, for both premiums and reserves, and go through the actual calculations. It would be well, at first, to take cases for which the correct result may be found among the tables included in this book. Logarithms may be employed to advantage in multiplications and divisions, but they are not necessary. In practical work by actuaries, logarithms have been largely supplanted by calculating machines, which perform the operations of multiplication and division satisfactorily, at a great saving of time and labor.

A very complete system of symbols has been devised and brought into general use, to denote algebraically the various values in use in life insurance calculations. In this book, however, it was thought best to use only the more elementary symbols, so that the study of the principles of insurance should not be complicated by the necessity of memorizing an extensive system of symbols. The use of symbols in this book will not, it is thought, cause any confusion for those who make a more advanced study of the science of insurance.



## CHAPTER XI.

## JOINT LIFE ANNUITIES AND INSURANCES.

NOTE (1):—If there be  $a$  chances of the happening of any event, that must either happen or fail to happen, and  $b$  chances of its not happening, then the probability of such event taking place will be represented by  $\frac{a}{a+b}$ . The probability of the event not happening will be  $\frac{b}{a+b}$ .

NOTE (2):—Since the above two fractions represent *all* the possible contingencies with regard to the event, their sum will indicate the probability of the event *either* happening or failing. This probability,—which is really a certainty,—will be represented by  $\frac{a}{a+b} + \frac{b}{a+b} = \frac{a+b}{a+b} = 1$ .

NOTE (3):—Since certainty of the happening or not happening of the event is denoted by 1, then, if the probability of the event's happening is known, the probability of its failure may be found by deducting the known probability from 1. In this case  $1 - \frac{a}{a+b} = \frac{b}{a+b}$ .

NOTE (4):—The probability of the concurrent happening of two or more events that are independent of each other is equal to the product of the probabilities of the happening of each event considered separately. The probability of failure in this case is found (under Note (3), above,) by subtracting such product from unity.

In all the previous statements of theory and formulas, it has been attempted to develop the various principles by general reasoning rather than by reference to the laws of probability. This was done because the theory of probabilities, though it would materially shorten the demonstration, is often found difficult of full comprehension, and might prove somewhat of a stumbling-block, at the beginning of their studies, to those who wish to learn something of life insurance principles without going

far into their intricacies. Now, however, that the reader has formed a general idea of insurance principles it will be a comparatively simple matter for him to understand the application of the theory of probabilities to insurance; so these theories will be used in the explanation of joint life contracts, which follows. We begin with the probabilities regarding a single life.

The mortality table states in its column of Number Living that out of a certain definite number of persons living at a particular age, a certain definite number will be living a year hence, without designating in any way the *identity* of those survivors. Continuing the explanation with a particular state of facts: At age 30 in the American Experience Table 85,441 persons are living, and a year later, at age 31, 84,721 persons of the original group are still alive. As each individual of the original group is equally likely to survive the year the mortality table shows that any man 30 years old, of the class of lives accepted as regular risks by insurance companies, has 84,721 chances out of every 85,441, of living a year. His probability of living one year is then represented by the fraction  $\frac{84721}{85441}$ .

The mortality table shows also that 84,000 persons out of the original 85,441 survive to age 32. In other words, there are for every man now 30 years old 84,000 chances out of 85,441 of living two years. His probability of living two years is then  $\frac{84000}{85441}$ . Similarly  $\frac{83277}{85441}$  is his probability of living three years, and so on, for each further year of life. (The application of the rule in Note (1) above is obvious.)

Stating the above algebraically, and assuming an age  $x$  we have as general expressions for the probabilities:  $\frac{l_{x+1}}{l_x}$ ,  $\frac{l_{x+2}}{l_x}$ ,  $\frac{l_{x+3}}{l_x}$ ,  $\frac{l_{x+4}}{l_x}$ , and so on. Then to find the value of an annuity of \$1 payable at the end of each year of life, we have only to multiply these values of the probabilities of living certain terms of years by the respective present values of \$1 payable "certain" at the close of those years, and sum the products. The result would be indicated by  $\frac{vl_{x+1}}{l_x} + \frac{v^2l_{x+2}}{l_x} + \frac{v^3l_{x+3}}{l_x} + \text{etc. to table limit}$ , or  $\frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc. to table limit}}{l_x}$ ,

which is the formula for the P. V. of an annuity with its first payment falling due a year hence.

JOINT LIFE ANNUITY:—We now seek a corresponding expression in which two lives are involved instead of one. The question is: What is the present value of an annuity to continue as long as two designated persons are *both* living? For convenience we designate this second life by  $y$ , and  $y$  may be equal to, greater or less than  $x$ . Then the corresponding probabilities of  $y$  living one, two, three, etc., years are  $\frac{l_{y+1}}{l_y}$ ,  $\frac{l_{y+2}}{l_y}$ ,  $\frac{l_{y+3}}{l_y}$ , and so on. We are not, however, seeking the value of an annuity during this life alone.

Applying the rule in the first part of Note (4), we find the probability that *both*  $x$  and  $y$  will be alive a year hence by multiplying together the individual probabilities. These are  $\frac{l_{x+1}}{l_x}$  and  $\frac{l_{y+1}}{l_y}$  and the product  $\frac{l_{x+1}}{l_x} \times \frac{l_{y+1}}{l_y}$  or  $\frac{l_{x+1} l_{y+1}}{l_x l_y}$  represents the combined probability.

By the same rule the probability of both lives continuing for two years is  $\frac{l_{x+2}}{l_x} \times \frac{l_{y+2}}{l_y} = \frac{l_{x+2} l_{y+2}}{l_x l_y}$ ; for three years it is  $\frac{l_{x+3} l_{y+3}}{l_x l_y}$ , and so on.

As payments of \$1 yearly under the annuity are to be contingent on the continued life of both  $x$  and  $y$  we may value the successive payments by multiplying the present values of \$1, payable certainly, one, two, three, etc., years in the future, by the fractions which express the probability of payment being made. The results would be  $\frac{v^1 l_{x+1} l_{y+1}}{l_x l_y}$ ,  $\frac{v^2 l_{x+2} l_{y+2}}{l_x l_y}$ ,  $\frac{v^3 l_{x+3} l_{y+3}}{l_x l_y}$ , and so on. The present value of an annuity for the joint life would then be  $\frac{v^1 l_{x+1} l_{y+1} + v^2 l_{x+2} l_{y+2} + v^3 l_{x+3} l_{y+3} + \text{etc. to the limit of table}}{l_x l_y}$ ,

Joint life commutation columns may be formed on the same principles as where one life only is involved;  $x$  is generally taken as the older life. By multiplying both numerator and denominator by  $v^x$  we have

$$\frac{v^{x+1} l_{x+1} l_{y+1} + v^{x+2} l_{x+2} l_{y+2} + v^{x+3} l_{x+3} l_{y+3} + \text{etc. to table limit}}{v^x l_x l_y},$$

or  $\frac{D_{x+1:y+1} + D_{x+2:y+2} + D_{x+3:y+3} + \text{etc.}}{D_{xy}}$ , or  $\frac{N_{x+1:y+1}}{D_{xy}}$ , which is denoted by  $a_{xy}$ .

The corresponding present value of a joint annuity-due is

$$1 + a_{xy} = \frac{N_{xy}}{D_{xy}}.$$

JOINT LIFE INSURANCE:—Under a contract of joint life insurance the company is bound to make payment of the sum insured at the failure of the *joint* lives, in other words, at the first death. When one of the lives insured fails, the sum insured becomes payable, and the contract is thereby terminated. The probabilities in this case are however not so readily seen as in the case of an annuity. We use as before the two lives  $x$  and  $y$ .

In the previous discussion we proved the probability, that both  $x$  and  $y$  would survive one year, to be  $\frac{l_{x+1}l_{y+1}}{l_xl_y}$ . Under the present assumptions, if  $x$ , or  $y$ , or both, die in the year, the sum insured is to be payable. Since these are the only other alternatives to *both* surviving the year the probability that the sum insured will become payable is equivalent to the probability of the *failure* of the original proposition, that both  $x$  and  $y$  live through the year. The desired probability therefore, by the rule in Note (3), above, is  $1 - \frac{l_{x+1}l_{y+1}}{l_xl_y}$

which may be algebraically transformed to  $\frac{l_xl_y - l_{x+1}l_{y+1}}{l_xl_y}$ . As the sum insured will, if to be paid at all, fall due a year hence, we find the present value or single premium for the insurance by multiplying  $v$ , which is the P. V. of \$1 payable certainly a year hence, by the above probability, the result being  $v \frac{l_xl_y - l_{x+1}l_{y+1}}{l_xl_y}$ .

Similarly the premium for one year's insurance at the joint ages  $x+1$  and  $y+1$  would be  $v \frac{l_{x+1}l_{y+1} - l_{x+2}l_{y+2}}{l_{x+1}l_{y+1}}$ , and so on for all higher ages.

It is to be noted that the probability  $\frac{l_xl_y - l_{x+1}l_{y+1}}{l_xl_y}$  is *not* equal to  $\frac{d_x}{l_x} \frac{d_y}{l_y}$ , though it may seem so at first glance.  $\frac{d_x}{l_x}$  represents the probability that  $x$  will die in the year, and  $\frac{d_y}{l_y}$  is the corresponding probability for  $y$ . Their product  $\frac{d_x}{l_x} \frac{d_y}{l_y}$ , according to the rule in Note (4), gives the probability that *both*  $x$  and  $y$  will die in the year.



The sum insured would, it is true, become payable if both should die in the year, but it would also be payable if only one of the two lives should fail, so this probability,  $\frac{d_x d_y}{l_x l_y}$ , does not cover all the contingencies under which the insurance would fall due.

To find the single premium for joint whole life insurance we must consider rather more complex probabilities after the first year of insurance. For the first year the probability, as before given, is  $\frac{l_x l_y - l_{x+1} l_{y+1}}{l_x l_y}$ . If either  $x$  or  $y$  dies in the first year the contract is terminated. Therefore we wish to find, for the second year of insurance, the probability that both *will* survive the first year but both *not* survive the second year. The first of these conditions is denoted by the expression  $\frac{l_{x+1} l_{y+1}}{l_x l_y}$ , and the second, under Note

(3) by  $1 - \frac{l_{x+2} l_{y+2}}{l_{x+1} l_{y+1}}$  or  $\frac{l_{x+1} l_{y+1} - l_{x+2} l_{y+2}}{l_{x+1} l_{y+1}}$ . Their product is

$$\frac{l_{x+1} l_{y+1}}{l_x l_y} \times \frac{l_{x+1} l_{y+1} - l_{x+2} l_{y+2}}{l_{x+1} l_{y+1}} = \frac{l_{x+1} l_{y+1} - l_{x+2} l_{y+2}}{l_x l_y}.$$

Similarly, the probability that both  $x$  and  $y$  will survive two years and one or both die in the third is

$$\frac{l_{x+2} l_{y+2}}{l_x l_y} \times \frac{l_{x+2} l_{y+2} - l_{x+3} l_{y+3}}{l_{x+2} l_{y+2}} = \frac{l_{x+2} l_{y+2} - l_{x+3} l_{y+3}}{l_x l_y}.$$

To find the single premium for joint whole life insurance we must multiply each of these probabilities, and others for later years, into the present value of \$1 payable certainly at the close of the year to which each applies, and find the sum. The denominators being  $l_x l_y$  in all cases, this takes the form:

$$A_{xy} = \frac{v(l_x l_y - l_{x+1} l_{y+1}) + v^2(l_{x+1} l_{y+1} - l_{x+2} l_{y+2}) + v^3(l_{x+2} l_{y+2} - l_{x+3} l_{y+3}) + \text{etc. to table limit.}}{l_x l_y}$$

Commutation columns for values of  $M_{xy}$  may be constructed along lines similar to those where one life is involved, and the formulas for joint life insurances are parallel to those for one life. The labor involved in making up such tables for every possible combination of ages is, however, so great that they are now generally formed only for the condition that  $x=y$ , that is, that the ages are equal. Values for contingencies involving lives of unequal ages can then be found on principles which are beyond the

scope of this book. Joint insurances on three or more lives are based on the same principles as for two lives.

The foregoing description of elementary joint life contracts will serve to indicate the possibilities of various combinations of probabilities on several lives, in complicated benefits involving the survivorship of one or more lives after the failure of others.

## CHAPTER XII.

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### LIFE INSURANCE ORGANIZATIONS.

THERE are in this country two general forms of life insurance organization. First are those usually known as "regular" life companies; also sometimes called "legal reserve," or "old line," companies. These offer to the public a purely business contract, to be paid for by fixed premiums computed upon scientific principles, and guaranteeing a definite amount of insurance. Their policies also provide explicitly for settlements with those who wish to abandon their contracts. In order that all these promises may surely be fulfilled, the regular companies are held to a strict accountability under the law, and required to be conservative.

The other type of organization is represented by what are known as "assessment companies," and "fraternal societies," of which a description will be given separately.

Regular life insurance companies may be divided into three classes, viz.:—first, those on the "mutual plan," second, those on the "stock plan," and third, those on the "mixed plan"—a combination of the other two. In a purely mutual company there is no capital stock, and therefore no stockholders. The business being owned by the policy-holders, the control of such a company, therefore, lies, theoretically at least, in the majority vote of the policy-holders. The advantage claimed for this form of organization is, that, as there is no stock, there are no dividends to be paid to stockholders and no interests to be subserved, other than those of the policy-holders. The practical difficulty which arises with this form of company is that it is often really impossible to obtain a true expression of the views of a majority of its policy-holders, many of whom refuse or neglect to exercise their right to vote. If, however, the persons in executive control of a mutual company are faithful to the trust placed in their care, the company may attain the conservative success which such management deserves. This has been proven in several cases.

A "stock" company, as its name implies, has capital stock, and in nearly all cases the control of such a company lies wholly with the

stockholders. In a strictly "stock" company, low premiums are charged and the policy-holders are not legally entitled to a share of the profits, as all the policies are issued on the non-participating plan, but just as the stockholders may allow the policy-holders to vote, they may also give them shares of the surplus gratuitously from time to time.

Companies on the "mixed plan" have some of the features of the other two plans. Though they have a capital stock, the control may be only partly in the hands of the stockholders, and the greater part of the surplus accruing from such companies' business goes to the policy-holders. Frequently the charters provide that the stock shall not be paid more than a certain rate of dividend, say seven per cent., and that all other surplus shall go to the policy-holders. Sometimes, however, the charters allow the stockholders to receive a certain fraction of the surplus accruing each year, in which case their dividends may become quite large, though not appreciably diminishing the dividends of the policy-holders. In some few cases there is no limitation whatever, and the stockholders may take such share of the surplus as they think proper, though they are legally bound by the policy contracts to make some dividends to such policy-holders as hold participating policies.

It is claimed on behalf of the "stock," and also for the "mixed" form of organization that the self-interest of the stockholders furnishes the best guarantee of the conservatism of the company, while competition with other companies will require able and economical management, combined with liberality toward the policy-holders.

It should be remarked that for many years past nearly all the purely mutual companies have to a certain extent issued low-rate non-participating policies, but they have been a very small fraction of the whole business of those companies and it is probable that after the year 1907, at the latest, no more will be issued by such companies.

These different classes of regular companies issue similar forms of policies, those of one class differing from those of another class only in the provisions regarding participation in surplus. The differences in administration are slight, and all are in general subject to the same laws.



Before going further it should be said that regular life insurance companies conduct either one, or both of two forms of life insurance, "Ordinary" and "Industrial;" differing, not in principle, but in many points of practical management.

In "Ordinary" insurance the premiums are stated on a basis of \$1,000 of insurance, which is usually the minimum amount issued in a single policy under this plan, and premiums are payable annually, semi-annually, or quarterly. This is the older form of insurance, and that taken by the well-to-do classes of people. "Industrial" insurance, as its name implies, is that taken by the working classes. It is issued for small amounts and is paid for by weekly premiums to suit the weekly wages. The premiums are almost always some multiple of 5 cents and the amount of insurance is adjusted thereto, more at young ages and less at old ages; thus "Industrial" premium tables will show that a man entering at the age of 25 will be given \$76 of whole life insurance for 5 cents a week, while a man aged 50 would get only \$35 of insurance. We will first discuss the characteristics of the practical management of Ordinary life insurance, and then describe more fully how Industrial insurance differs from it.

## CHAPTER XIII.

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### PREMIUMS AND POLICY PROVISIONS.

GROSS PREMIUMS:—The premiums which we have been discussing in the previous chapters have all been “net;” that is, they have been simply the mathematical equivalents of the insurances to which they applied, no provision being made for any practical considerations, such as expenses and contingencies other than those connected with mortality.

Like any other business, however, life insurance is subject to expenses of management, which must be borne by the persons to be insured. Chief among the expenses are the commissions which must be paid the agents of the company for obtaining applications for insurance in the first place, and then for collecting the premiums on the policies after they are issued. Besides this, there are such expenses as are common to any corporation—salaries of officers and clerks, rental of offices, taxes, advertising, and an indefinite number of other items which come up in the course of a company’s business.

Life insurance companies are also liable to contingencies of many kinds not peculiar to their business. Thus, for example, there may be losses from unfortunate investments or from defalcations by agents.

To provide for the expenses above stated and against the contingencies just mentioned the companies add to the “net” premiums certain amounts called “loadings” or “margins.” These two parts, taken together, make up the “gross” or “office” premiums, or “rates,” which are to be collected from the insured. Commissions to agents, taxes and various other expenses can often be conveniently expressed as a percentage of the gross premium, and for this reason the “loading” or “margin,” which is added to the net premium to form the gross premium, frequently is made a percentage of the net premium. A simple percentage loading will be greater in amount at a high age of issue than at a lower age. Often such a percentage is supplemented by a fixed addition, the same at each age. This tends to make the

total loading at a low age greater in proportion, though less in amount, than at a higher age, and there are good practical reasons for this arrangement, in the fact that certain expenses are the same no matter what the age of the insured. Another common system of loading premiums is to add to the net premium a certain percentage of itself and also a percentage of the net premium at the same age for an ordinary life policy. This generally results in making the loading at higher ages greater in proportion as well as in amount than at lower ages for the same form of policy, which is objectionable.

The gross premiums charged for insurance of \$1,000 or over are always directly proportioned to the amount of insurance, though it might be reasonable to reduce the loading in the premium on a policy for a large amount. In determining this question of loading for premiums, matters of principle often have to give way to practical considerations and legal requirements, and no system has yet been devised which is perfectly satisfactory in all respects.

The amount of the loading is very largely determined by the question whether the policy is to be participating or non-participating. In the case of participating insurance, to ensure safety, the premiums are made larger than is considered absolutely necessary, and the company engages to return to the insured such portion of the funds paid in to the company as in the judgment of the officers and directors is in excess of the company's needs, and can safely and equitably be returned to him. The amount of such return is, however, not guaranteed, and the insured runs the risk of receiving back in "dividends," as they are called, very little or even no part of the money he has paid in.

In the case of non-participating business, the insured pays a premium containing a smaller loading for expenses and contingencies, and has no right to a return of any savings made in the business of the company. The premiums in this case are always smaller than the participating rates, but cannot be less than the "net" premiums. Thus where the gross annual premium charged for \$1,000 of participating whole life insurance at age 40 is from \$31 to \$33, the corresponding non-participating rate is about \$27, the net premium in this case being about \$24. In other words, there is a margin of from \$7 to \$9 in the participating rate and only \$3 in the non-participating.

It is not to be supposed, however, in the case of non-participating business that a company depends solely on the small loading in the premium for expenses and profits. It expects to profit from earning a higher rate of interest than was assumed in the calculations on which its premiums and reserves are based, and also from a mortality cost less than provided for in the net premiums. When a stock company issues non-participating policies it exposes its capital and surplus to the risk that expenses and losses of all kinds will prove greater than receipts. When a mutual company issues such a policy it risks in the same way whatever surplus funds may remain undistributed to policy-holders.

**ANNUAL, SEMI-ANNUAL, AND QUARTERLY PREMIUMS.**—Ordinary life insurance is based on the assumption that premiums will be paid “annually,” or in other words, that the premium for each policy year will be paid in one sum at the beginning of the year; but in practice, on account of the size of the premiums which would thus be payable, the insured is given the option of paying the premiums instead, in semi-annual or quarterly instalments. The semi-annual instalment is generally calculated by adding four per cent. to the gross annual premium and taking one-half of it. The quarterly premium is usually formed by adding six per cent. to the annual rate and taking one-quarter. This large additional loading in each case is partly for the purpose of making up for the loss of interest caused by not receiving the premium in one sum at the beginning of the year, but more particularly to cover the extra cost of collection, and the risk that the insured will neglect to pay the rest of the premium. If the insured dies before he has paid all of the instalments corresponding to the annual premium for the year, the amount of those instalments, called the “deferred premiums,” is deducted from the amount of his policy.

**PREMIUMS PAID BY NOTES:**—Besides payment in instalments, premiums are often paid in part by notes. By this system, only a portion of an annual premium is paid in cash at the date when payment of the whole is due, the remainder being covered by notes payable at some stated times later in the policy year. In such cases many companies collect the interest in advance and others allow it to be paid along with the notes. The continuance of the policy in force is then dependent on the settlement of these notes at their due dates. This system is more economical to the insured than payment by semi-annual or quarterly premiums.



Should any notes be outstanding at the maturity of the policy their sum, with interest, is deducted from the principal sum payable.

**AGE OF THE INSURED:**—In the explanation of the theory of the business, we always assumed that the insured was exactly of a certain age, as 20 or 30. In practice, however, very few policies are issued on the birthdays of the insured, and he is “rated,” i. e., charged the premium for, his age according to his nearest birthday, an exact half year counting as a year of age in this connection. In this way some are counted younger and others older than they are, but the ages are thus averaged so as to agree as closely as necessary with the actual facts. The above is the practice in the United States, but elsewhere it is usual to “rate” according to the age *next* birthday.

**BENEFICIARY:**—This is the name given to the person to whom the insurance is payable. Under most forms of policy it is immaterial who this person, or these persons,—for there may be several,—may be, except that he or they should be properly qualified by relationship, or business connection, to receive the insurance.

The general rule is that they must have an “insurable interest;” that is, they must bear to the insured such a relation that they would suffer, by his death, some appreciable financial loss, actual or contingent; but there are exceptions to this rule. Sometimes a policy is made payable to “the estate of the insured,” in which case the insurance would be paid as directed in his will. Policies generally contain a provision giving the insured an unrestricted right to change the beneficiary.

**PAYMENT OF CLAIM:**—When a policy matures by the death of the insured, payment of the sum insured is not made until satisfactory proof is given that the death was really that of the insured, and that his death did not result from some of the causes against which the company has refused to give insurance. These facts may be shown within a day or two, or proofs may not be submitted for several months, or even years. Most companies stipulate that they will make payment of the “claim” as soon as satisfactory proofs are given; others promise to pay within sixty or ninety days thereafter. This practice of making payments soon after the death is a slight departure from the assumption made in life insurance theory, that a death claim falls due only at the close of the policy-year in which the death occurs.

Death claims, however, are not always paid in one sum at the insured's death, for most policies now issued provide that payment may be made at the option of the insured in equitable instalments extending for a term of years, or even during the life of the beneficiary. This arrangement serves to give the desired insurance protection to the beneficiary without risking the loss of the entire sum by unwise investment. As these instalment arrangements are merely modes of settlement equivalent to payment in one sum in the year of death, it is usual to consider that the full amount is disbursed as a death claim and then received again as a sum to be held in trust to be paid out in instalments.

**POLICY CONDITIONS:**—In the early history of life insurance, when the business was still regarded as something of an experiment, the policy offered was a hard and fast contract, with many severe conditions and few, if any, privileges. The insured was required to pay the premium on or before a fixed date, without days of grace. He was greatly restricted as to residence or travel, and as to the occupations in which he could engage. If he violated any of these rules his policy was absolutely forfeited, no matter how long he had been paying premiums, and if any allowance was made him, it was simply by the grace of the company.

As time passed, and the business came to be better understood by its managers and the public, the needlessness and inequity of this extreme conservatism became evident, and at present the tendency is to make the provisions of a policy more liberal than is entirely safe for the company. A month's grace is now allowed the insured in payment of premiums, and very slight restrictions are imposed as to residence and occupation. If he fails to pay a premium when it falls due he may have his policy restored to full force if he complies with some reasonable requirements within a certain time.

The "Incontestability" provisions of the policies of nearly all companies preclude the companies from making any defense against a death claim under a policy that has been one or two years in force if all premiums have been duly paid. Thus no matter how misleading the statements made by the insured in his application, or how great the special hazard to which he may expose himself by changing his occupation or residence, the company must pay if he dies. In some few cases policies are made incontestable even from date of issue.

**LOANS AND SURRENDER VALUES:**—Under present conditions, after premiums have been continued for two or three years the insured may borrow from the company on the security of his policy to pay premiums, or for other purposes; or he may surrender his policy and receive an equitable allowance of cash or its equivalent in insurance. Moreover, the amounts thus available are guaranteed by the company and definitely stated in the policy itself.

The basis of these loans and surrender values, as they are called, is the reserve for the policy; and it was not until the true character and purpose of reserves were understood that these features came into general use. Many States now have what are called non-forfeiture laws, requiring the companies to allow on demand a certain minimum amount of insurance based on the amount of reserve held for a policy at the time it is surrendered or lapses for non-payment of premiums; the companies, however, usually allow much more than is required by law. The amount by which the allowance thus made is short of the full reserve for the policy is called the "surrender charge."

The considerations leading to the allowance of these surrender values are in general as follows:—The insured has been paying the company the stipulated gross annual premium, containing the net premium with a certain loading. This premium in the early years of the policy has been more than sufficient to pay for the insurance given in those years, and the company has been setting aside and accumulating these excesses in the net premiums, in order to have sufficient funds on hand on account of the policy to meet the cost of insuring the man when, later in life, the yearly cost of the insurance exceeds the yearly net premium. So, if the insured terminates the contract by surrender or by failing to pay a premium when it falls due, the company is thereby relieved from any future liability to give insurance on his life, and these accumulated excesses of net premium—the reserve—are no longer needed by the company for the purpose for which they were intended. Though the insured has broken his contract, it has been found that it is not necessary to penalize him by refusing to allow him any further benefit from his payments, in order to protect the other party to the contract—that is, the insurance company, or the other policy-holders that keep their policies in force. Just how much a company may allow a retiring policy-



holder, with safety and equity towards the company or those remaining in it, is, however, a matter of opinion, depending on many practical considerations.

As the expenses connected with the issue of a policy are much greater than the margin for expenses in the first year's premium, it may be that up to the time of surrender the company has expended, in connection with this particular policy, considerably more than the margins in the premiums already received, depending for reimbursement on the margin in the future premiums which the insured has agreed to pay under the contract. Obviously, to allow a retiring policy-holder in such a case the full theoretical reserve, would not be just to the remaining policy-holders, who are fulfilling their contracts to pay their premiums, and if the full reserve were allowed him on surrender they would in the end be forced to make good to the company part of the sums expended on account of his policy.

A second reason for making the surrender allowance less in value, or in amount, than the full reserve held for the policy, lies in the danger that the exercise of the options of obtaining a loan or a cash surrender value by a large number of its policy-holders during hard times may involve financial loss to the company. Savings banks, with which life insurance companies have some points in common, usually reserve the right to defer payment, at their option, of all but a small part of the sums deposited with them, but life companies ordinarily make no such restriction. It is desirable for all persons connected with an insurance company that its assets shall be invested at the best rates of interest consistent with safety. This often involves the selection of investments which, though safe, are not readily convertible into cash. In times of business depression, such as this country has seen more than once, even the best securities will suffer serious depreciation though their certainty of payment remains unquestioned. Such a financial crisis is just the time when policy-holders, in need of cash, are most likely to demand surrender values from the company, thus not only reducing its premium income, but also forcing the sale of securities at less than their true value, and perhaps crippling the company. In such a case the persons exercising these options should properly not be allowed a greater proportion of the reserves on their policies than the company is able to realize on the true value of its securities sold to provide cash for retiring



policy-holders. This matter, however, cannot be regulated by any set of rules, but depends on the amount of the company's assets, the character of its business and investments, and the form of its organization.

Another consideration is more technical in character, and its importance is somewhat in dispute. When an insured man gets into poor health, or contracts some incurable disease, his insurance becomes of great value in his eyes, because he realizes that his death may be imminent, when the policy would perform its beneficent office; and such a man will make sacrifices to pay his premium and keep his policy in force. If, on the other hand, a man is in excellent health and the payment of a premium involves some hardship, he does not realize so fully the value of the insurance, and will not hesitate so much about letting the policy lapse. It is thus argued that if surrender values are very liberal the result would be as follows: practically all the lapses would be those of persons who were in good health and did not feel the need of insurance, very few who were in bad health leaving the company, so that the average vitality of the persons remaining in the company would be diminished. Therefore, in determining what surrender values shall be allowed retiring policy-holders something should be deducted from the reserves to provide for the higher mortality to be expected among those who remain. As before stated, the correctness of this theory is sometimes questioned. It is extremely difficult to obtain data on which to base conclusions in this matter, but some statistics seem to show that when a man in poor health thinks of allowing his policy to lapse, he either does not appreciate his condition, or does not take it into consideration as fully as he might reasonably be expected to do.

When a policy is surrendered, the insured generally has three options of settlement. He may accept the cash surrender value which is guaranteed him in his policy, and terminate all connection with the insurance company. He may elect what is known as the "extended insurance" or "continued insurance" option, in which case the cash value is used as a single premium to purchase term insurance, for the full amount of the policy, during as long a period as it will pay for. At the expiration of such a term the policy has no further value. Under the third option, "paid-up insurance," the insured can have the policy's cash value used as a single premium to buy fully paid insurance of the same

form as the original policy. In this case it may be considered that part of the original policy is made to become "paid-up" and the rest is discontinued.

Most policies have what is often termed an "automatic non-forfeiture provision," by which, if the insured fails to exercise his right of selection within a certain period after the lapse of his policy, either the second or the third option above indicated becomes operative without action on his part.

**POLICY PLANS:**—It may be well here to review the various kinds of policies usually written by regular insurance companies, showing their several advantages and disadvantages from the viewpoint of the person desiring insurance.

**ORDINARY WHOLE LIFE INSURANCE BY CONTINUED PREMIUMS:**—This form of policy guarantees the payment of the principal sum at the death of the insured, and requires payments from him every year during his life. It is the cheapest form of policy by which a man can insure for a fixed sum in return for a premium which remains the same during his life. Its objection, however, lies in the fact that the insured cannot count on getting through paying premiums.

**LIMITED PAYMENT WHOLE LIFE INSURANCE POLICY:**—This form of policy, as its name implies, gives the same insurance as the previous one, but the premiums are payable only for a term of years, the insurance being continued thereafter without further payments. The term to which premiums are limited can be made as long or as short as is desired, the premium increasing as the term is shortened. The term that is selected in a very large majority of cases is 20 years, making the policy that is styled a 20-payment life policy. This is, in fact, the most popular form of policy.

**ENDOWMENT POLICY:**—This form of contract provides that the amount of the policy shall be payable at the death of the insured at any time during a certain period, and if he survives the period, shall be paid to himself. This policy may be issued to mature at a certain age of the insured, or at the close of a certain period. It is, however, generally issued to mature at the end of 20 years, being what is known as the 20-year endowment policy. This form of policy is next in popularity to the 20-payment life policy, and calls for a considerably larger premium at the same age.

Some companies issue policies which are to mature as endowments at some advanced age, such as 80 or 85, and cost but little more than whole life policies. This form of policy is offered because it is argued that insurance beyond that advanced age is in most cases of little practical use compared with the advantage of having the policy mature as an endowment, and thus provide the policy-holder with a fund for his maintenance during the rest of his life.

**TERM INSURANCE POLICY:**—A term insurance policy provides for payment of the amount insured only in case the insured die within a certain limited term. At the close of that term the insurance ceases and ordinarily no return is made on account of the payments received from the insured. This sort of life insurance may be fairly compared with ordinary fire insurance; where if a man insures his house for one or more years, he never thinks he should receive any return of premium after the policy has expired, for he understands that he has had the value of his money in the insurance enjoyed. The terms for which this form of policy is commonly issued are 1, 5, 10, 15, and 20 years. The premiums, owing to the limited period for which insurance is given, are quite low in comparison with most other forms. The cheapest, but not really the most economical form of insurance, is the one-year term insurance, giving insurance for one year only. It is usual in connection with term policies to give the insured the right at the end of any term to renew his policy for a like term, paying an increased premium corresponding to his increased age. It is obvious that, while in the earlier years of such a policy the premiums would be very low in comparison with other forms, in the later years of a long life the premiums would become so high as to be prohibitive, particularly in the case of one-year renewable term insurance, so companies generally require that some modification shall be made in the contract when an advanced age, such as 70, is attained. The term plan of insurance is adapted for use in cases where, for business or other reasons, it is known that the necessity for insurance will be only temporary. The loading in term premiums is relatively quite large, and particularly so for participating policies, and in general no surrender values are guaranteed.

**RETURN PREMIUM POLICIES:**—Another form of policy somewhat issued is the Return Premium Policy, described on page 62,



providing that if death occur during a certain limited period all premiums which have been received on the policy up to that time shall be paid in addition to the principal sum insured. This form gives increased insurance and calls for a premium slightly larger than for the corresponding policy without return of premium.

**SPECIAL FORMS:**—Besides the forms of policy above described, there are many special plans issued by various companies. These are in general combinations of some special feature with one of the above mentioned forms. It may be taken as a general axiom, that, if a policy offers any special advantages in addition to the benefits included in the regular form, the company must obtain additional payment from the insured for these additional features, either by increased premiums, reduced values on surrender, or the declaration of smaller “dividends.” Conversely, if the premium charged for a special form of policy is smaller than usual it will be likely to be found that the actual insurance or other benefits offered are in some way less than generally given for the larger premium.

It is not implied, however, that all special policies conceal some “catch,” though that is often the case, for some such forms serve a most excellent purpose, notably those providing that the sum insured shall be paid, not in one sum, but in yearly instalments covering a period of years or the life of the beneficiary.

**SPECIMEN PREMIUMS:**—Below are given, for practical illustration and comparison, average participating premiums per \$1,000 insurance at various ages, for some of the more usual plans of insurance just outlined.

*Participating Premiums on Various Plans.*

Age.	Ordinary Whole Life.	20 Pay- ment Life.	15 Pay- ment Life.	10 Pay- ment Life.	1 Year Term.	5 Year Term.	Age.
25	\$20.65	\$29.30	\$35.20	\$47.00	\$11.20	\$11.85	25
30	23.55	32.25	38.65	51.50	12.25	12.70	30
35	27.30	35.80	42.75	56.75	13.70	13.90	35
40	32.25	40.30	47.85	63.20	15.50	15.70	40
45	38.80	46.00	54.00	70.70	18.40	18.60	45
50	47.75	53.70	62.05	80.10	23.75	24.00	50
55	60.05	64.25	72.60	91.70	31.75	32.50	55



*Participating Premiums on Various Plans—continued.*

Age.	10 Year Term.	15 Year Term.	20 Year Term.	10 Year Endow- ment.	15 Year Endow- ment.	20 Year Endow- ment.	Age.
25	\$12.00	\$12.40	\$13.00	\$103.05	\$65.75	\$47.70	25
30	13.25	13.90	14.70	103.60	66.45	48.45	30
35	15.00	15.95	17.30	104.30	67.35	49.60	35
40	17.60	19.35	21.50	105.55	68.85	51.50	40
45	21.90	24.70	28.10	107.25	71.05	54.40	45
50	29.10	33.55	38.10	110.35	75.15	59.45	50
55	40.60	46.60	52.70	115.60	81.75	67.60	55

SPECIMEN POLICY:—As the main characteristics of the policies now in use have been explained it will be interesting at this point to examine a form of policy contract such as is actually issued. For this purpose there is given below an Ordinary Whole Life policy for \$10,000 issued to John Sample, who is 40 years of age, in favor of his wife Mary, at an annual premium of \$322.50. The form used is that prescribed in 1906 by the New York Legislature for use by the companies of that State after January 1st, 1907.\*

Amount, \$10,000.

Age 40.

Annual Premium, \$322.50.

### THE STANDARD MUTUAL LIFE INSURANCE CO.

IN CONSIDERATION of the.....annual premium of *Three Hundred Twenty-two Dollars and Fifty Cents*, and of the payment of a like amount upon each *First* day of *March* hereafter until the death of the Insured,

PROMISES TO PAY at the Home Office of the Company in New York upon receipt at said Home Office of due proof of the death of *John Sample* of *New York*, County of *New York*, State of *New York*, herein called the Insured, *Ten Thousand Dollars*, less any indebtedness hereon to the Company and any unpaid portion of the premium for the then current policy year, upon surrender of this Policy, properly receipted, to *Mary Sample, wife of the Insured*, beneficiary, with..... right of revocation.

\*Italics are used to denote matter which would, in practice, be written into the policy, and phrases or sentences not expressly prescribed in the State standard form are enclosed in brackets.

**CHANGE OF BENEFICIARY:**—When the right of revocation has been reserved, or in the case of the death of any beneficiary under either a revocable or irrevocable designation, the Insured, if there be no existing assignment of the Policy made as herein provided, may, while the Policy is in force, designate a new beneficiary with or without reserving right of revocation by filing written notice thereof at the Home Office of the Company, accompanied by the Policy for suitable endorsement thereon. Such change shall take effect upon the endorsement of the same on the Policy by the Company. If any beneficiary shall die before the Insured the interest of such beneficiary shall vest in the Insured.

**PAYMENT OF PREMIUMS:**—The Company will accept payment of premiums at other times than as stated above, as follows:

*In semi-annual instalments of \$167.70 payable on the first day of March and September, or in quarterly instalments of \$85.50 payable on the first day of March, June, September and December.*

Except as herein provided the payment of a premium or instalment thereof shall not maintain the Policy in force beyond the date when the next premium or instalment thereof is payable.

All premiums are payable at said Home Office or to any agent of the Company upon delivery, on or before date due, of a receipt signed by an Executive Officer, [the executive officers are,—the President, Vice-Presidents, the Secretary and the Treasurer.] of the Company and countersigned by said agent.

A grace of thirty days subject to an interest charge at the rate of [six] per centum per annum shall be granted for the payment of every premium after the first year during which time the insurance shall continue in force. If death occur within the days of grace the unpaid portion of the premium for the then current Policy year shall be deducted from the amount payable hereunder.

**CONDITIONS:**—[In the event of the death of the Insured within one year from the date hereof, by his own hand, whether sane or insane, or in consequence of his own criminal act, the liability of the Company on this Policy shall be limited to an amount equal to the premiums paid hereon.]

**INCONTESTABILITY:**—This policy shall be incontestable, except for non-payment of premiums [after one year] from its date. If the age of the Insured has been misstated, the amount payable hereunder shall be such as the premium paid would have purchased at the correct age.

**PARTICIPATION:**—The proportion of the surplus accruing upon this Policy shall be ascertained and distributed annually and not otherwise.

**DIVIDENDS:**—Dividends at the option of the owner of this Policy shall on the first day of March of each year be either—

- (1) Paid in cash; or,
- (2) Applied toward the payment of any premium or premiums; or,
- (3) Applied to the purchase of paid-up additions to the Policy; or,
- (4) Left to accumulate to the credit of the Policy with interest at [three] per centum per annum and payable at the maturity of the Policy, but withdrawable on any anniversary of the Policy.

Unless the owner of this Policy shall elect otherwise within three months after the mailing by the Company of a written notice requiring such election the dividends shall be applied to purchase paid-up additions to the Policy.

**LOANS:**—The Company at any time will advance upon the sole security of this Policy, at a rate of interest not greater than [six] per centum per annum, a sum not exceeding the amount specified in the table of loans set forth, deducting therefrom all other indebtedness hereon to the Company. Failure to repay any such advance or interest shall not avoid this Policy unless the total indebtedness hereon to the Company shall equal or exceed the aggregate of all unpaid dividends and accumulations and of [eighty] per centum of the net value of the Policy and all additions thereto, and thirty days' notice shall have been given by the Company.

**ASSIGNMENT:**—No assignment of this Policy shall be binding upon the Company unless it be filed with the Company at its said Home Office. The Company assumes no responsibility as to the validity of any assignment.

**OPTIONS ON SURRENDER OR LAPSE:**—After this Policy shall have been in force three full years it may be surrendered by the owner at any time prior to any default or within three months after any default. Thereupon,

(1) If there be no indebtedness hereon to the Company, the owner may elect either (a) to continue the insurance in force for its face amount and any outstanding dividend additions, but without future participation, and without the right to loans; or, (b) to purchase non-participating paid-up life insurance payable at the same time and on the same conditions as this Policy. The periods for which the insurance will be continued and the amounts of paid-up life insurance which will be allowed, exclusive of the application of dividend additions, are shown in the table of surrender values herein set forth.

#### TABLE OF LOAN AND SURRENDER VALUES.

The loan and paid-up insurance values stated in the following table apply to a Policy for \$1,000. As this contract is for \$10,000 the loan or paid-up insurance available in any year will be *ten times* the amount stated in the table for that year.

The period of paid-up continued insurance remains the same for a Policy of any amount.

After Policy has been in force.	Loan Value.	Paid-up Life Insurance.	Paid-up Continued Insurance.		
			Years.	Months.	Days.
3	\$.....	\$.....	.....	.....	.....
4	\$.....	\$.....	.....	.....	.....
*	* .....	* .....	*	*	*
(No figures	are here enter	ed, because the	values allowed by	different	
Companies	may vary cons	iderably. It is	the usual practice,	however,	
to allow	more than req	uired by law.)			
*	* .....	* .....	*	*	*
19	\$.....	\$.....	.....	.....	.....
20	\$.....	\$.....	.....	.....	.....
Years					



Values for later years will be computed on the same basis and be furnished upon request.

(2) If there be any indebtedness hereon to the Company, it shall be deducted from the amount which otherwise would be applicable as a surrender value to the purchase of temporary insurance for the period aforesaid, and the owner may elect either to have the remainder applied (a) to continue the insurance in force without participation and without the right to loans for the face amount of this Policy and dividend additions, less the indebtedness; or (b) to purchase a proportionate amount of non-participating paid-up life insurance.

If in the event of any default in the payment of premium or otherwise, after the Policy shall have been in force three full years, the owner shall not exercise either of said options within three months after such default, the insurance shall be continued as provided by option (a) in either paragraph (1) or (2).

In any case of continued temporary insurance under any of the above provisions this Policy, upon evidence satisfactory to the Company of insurability, may be reinstated within the first three years of the term for which the insurance is continued by payment of arrears of premiums and of whatever indebtedness hereon to the Company existed at the date of surrender or default, with interest at a rate not exceeding [six] per centum per annum.

**MODES OF SETTLEMENT:**—The Insured or the owner, or the beneficiary after the Insured's death, in case the Insured shall have made no election, may by written notice to the Company at its Home Office, elect to have the net sum payable under this Policy upon the death of the Insured paid either in cash or as follows:

(1) By the payment of an annuity equal to [three] per centum of such net sum payable at the end of each year during the lifetime of the beneficiary, and by the payment upon the death of the beneficiary of the said net sum, together with any accrued portion of the annuity for the year then current, unless otherwise directed in said notice, to the beneficiary's legal representatives or assigns.

(2) By the payment of equal annual instalments for a specified number of years, the first instalment being payable immediately, in accordance with the following table for each one thousand dollars of said net sum.

(3) By the payment of equal annual instalments payable at the beginning of each year for a fixed period of twenty years and for so many years longer as the beneficiary shall survive, in accordance with the following table for each one thousand dollars of said net sum.

Any instalments payable under (2) or (3) which shall not have been paid prior to the death of the beneficiary shall be paid, unless otherwise directed in said notice, to the beneficiary's legal representatives or assigns.

When any option calling for annual payments is elected, this Policy shall be surrendered upon its maturity and a supplementary non-participating contract shall be issued for the option elected.

Unless otherwise specified by the owner or by the beneficiary in making such election, the beneficiary may at any time surrender the contract guaranteeing the payment of instalments, for the commuted value of the payments



yet to be made, computed upon the same basis as option (2) in the following table; provided that no such surrender and commutation will be made under option (3) except after the death of the beneficiary occurring within the aforesaid twenty years.

TABLE OF INSTALMENTS FOR EACH \$1,000.

OPTION (2).				OPTION (3).			
Number of Annual Instalments.	Amount of Each Instalment.	Number of Annual Instalments.	Amount of Each Instalment.	Age of Beneficiary at death of Insured.	Amount of Each Instalment.	Age of Beneficiary at death of Insured.	Amount of Each Instalment.
(The	amount	s here en	tered by	companie	s may di	ffer consi	derably.)
* *	* *	* *	* *	*	* *	*	*

¶ No person except an Executive Officer of the Company as aforesaid has power to modify, or in event of lapse to reinstate, this Policy or to extend the time for paying a premium.

IN WITNESS WHEREOF, the Company has caused this Policy to be executed this *First day of March, 1907.*

*Henry Sample,*  
Secretary.

*William Sample,*  
President.

A LIMITED-PAYMENT LIFE policy-form would differ from the above only in respect to a clause limiting premium payments to a specified number of years, the premium being higher than quoted above.

A TWENTY YEAR ENDOWMENT policy-form would have similar limitations as to the payment of premiums which would be even higher than for the last form. Its second paragraph would read as follows:—

“PROMISES TO PAY at the Home Office of the Company in New York to *John Sample of New York, County of New York, State of New York*, herein called the Insured, on the *First day of March* in the Year *Nineteen Hundred and Twenty-Seven*, if the Insured be then living, or upon receipt at said Home Office of due proof of the prior death of the Insured, to *Mary Sample, Wife of the Insured*, beneficiary, with . . . . .right of revocation, *Ten Thousand Dollars*, less any indebtedness hereon to the Company and any unpaid portion of the premium for the then current policy year upon surrender of this Policy properly receipted.”

Besides these differences there would be another in the table of surrender values, where in many cases there would be a provision for a cash payment in case the Insured should outlive the "Continued Insurance" term; and there would also be several more "Modes of Settlement."

A copy of the application, upon which the insurance is based, is very often either written in or attached to the policy.

The policy-forms here given provide for an annual distribution of surplus, a subject which will be considered in the following chapter.

It should be stated that these forms are not given as models of clearness and general excellence, but because so many similar policies will probably be issued to the large business done by the New York companies. Several companies, not subject to the laws of the State of New York, have much better and clearer forms, which can be easily understood by anyone.

The clumsiness of the New York standard form is generally acknowledged, and, as it is positively obligatory *only* where policies are issued by New York companies to citizens of New York, some of the New York State companies have decided to use a better form for policies sent outside of that state.

## CHAPTER XIV.

## DIVIDENDS.

THE term "dividends" as used in life insurance has quite a different meaning from that attached to the more proper use of the word. Dividends, properly speaking, are derived from the earnings of invested capital, but life insurance dividends are essentially simply such portions of the funds received from the insured as a company's managers consider may be safely given back to them, with interest. Thus these so called "dividends" are ordinarily merely the return of excesses of payments, or of sums saved from the fact that the business has been carried on for less money than was collected to maintain it.

It should be remarked, however, that there was a time in the early days of life insurance when dividends were in part truly "profits." The expenses were then very light and more than offset by the gains from lapses, so that those who kept their policies in force made a direct profit from the losses of those who discontinued and received little or nothing in the way of surrender value. In that way the dividends to persisting policies in those times were larger than the surplus resulting from their own gross payments; but now, and for very many years past, owing to the high cost of obtaining new business and the liberal allowances to retiring policy-holders, the expenses always exceed the profit from discontinuances, and the yearly dividends now declared are in no true sense "profits," but only returns of surplusage in the premium.

THE SOURCES OF DIVIDENDS:—If a company assumes that it will earn three per cent. per annum from the investment of its funds, and bases its premiums and reserves on that assumption, but actually earns four per cent. per annum, that one per cent. of difference is in excess of what is necessary to the solvency of the company and may be turned over to those who contributed to the funds, if deemed advisable. This is an example of what is termed the "surplus from interest."

Life insurance companies in this country find that on the average they do not experience as heavy mortality as is provided for in their premiums. Thus, if a company's calculations were based on the American Experience Table, and it had at risk at the beginning of a year \$69,804 in one year term policies on the lives of 69,804 persons fifty years old, it would expect from the table that 962 would die during a year from that time, requiring payment of \$962 on account of those deaths. If, however, only 90 per cent. of that number, or 866, died, calling for the payment of \$866, there would thus result a saving due to the difference between the "expected mortality" and the "actual mortality" of \$96, or 10 per cent. of the "expected." This is termed a "saving from mortality." Too much reliance should not be placed on the showing in this respect for any one year, as the "actual mortality" may be less than the "expected" in one year and greater than the "expected" in the next, and the only safe way is to find the average experience of the company for a number of years.

Another source of dividends lies in the fact that the actual expenses may be found to have been less than the margins or "loadings" in the gross premiums, which were provided to meet expenses and contingencies. In the present state of business, however, but little surplus results from this source, except among the companies that are very economically managed.

Sometimes surplus is considered to result from the fact that, when policies are lapsed, all or part of the reserves on those policies are retained by the company free from further liability. As life insurance is at present conducted, however, with large surrender values and high expenses, this can hardly be counted as a source of profit or "saving," and any advantage derived by companies in this way is generally considered only as an offset to expenses.

Dividends are generally apportioned on what is known as the "Contribution Plan," or some variation of that system. The Contribution Plan seeks to apportion to each policy such share of the company's total divisible surplus as has been *contributed* by that policy. By this plan each policy is credited with the terminal reserve at the end of the previous year, and the annual premium actually paid less the expenses chargeable thereto, and also with the net interest earned on the sum of these items: it is



debited with the estimated actual cost of the insurance for the current year in its own case, and also with the terminal reserve which the company must hold on its account at the close of the current policy year. The balance will be the surplus which may be considered to have been contributed by that policy. If the sum of the estimated "contributions" for all policies practically agrees with the amount of surplus which the company considers properly divisible, each policy would be given a dividend equal to its "contribution," but if the aggregate contributions are greater or less than the total amount to be divided, each policy is given a correspondingly less or greater share of the surplus. This system may be varied so as to apply to an apportionment for a term of years instead of one year.

**ANNUAL DIVIDEND DISTRIBUTION:**—Some policies provide that dividends shall be payable at the end of the first or second year of the policy and annually thereafter. Such dividends are known as "Annual Dividends," and commonly the allowance of an annual dividend is conditioned on the payment of the premium for the next year following. Such dividends may be used by the policy-holder for the purchase of additional insurance payable with his policy, or may be applied in payment of premiums or other indebtedness. In the first mode of application, if a man aged 45 had a cash dividend of, say, \$25, he could have his policy increased by a "reversionary addition" of about \$50, which would be paid-up and payable with the policy. In well-established companies the annual dividends generally increase gradually year after year, because as the reserves are increasing the surplus from extra interest usually increases also.

**DEFERRED DIVIDENDS:**—When policies provide that no dividends shall be payable until the close of a period of years, such dividends are known as "deferred dividends." This period is usually 5, 10, 15 or 20 years, the most common period being 20 years. Policies having such provisions regarding dividends are known by such names as "20 Year Accumulation Policies," "20 Year Distribution Policies," "Semi-Tontine Policies," or similar terms. The principal element of this system is that those policy-holders who continue payment of premiums to the close of the dividend period receive all the dividends which under the annual dividend system would have gone to those policy-holders who by reason of death or lapse failed to continue payment to the end of

the period. This system, however, works no necessary injustice to those who thus fail to receive dividends, for the proviso as to distribution is a clear matter of contract known to the insured at the issue of the policy.

The deferred dividend system is not to be confused with the forms of "Tontine" policies which were in vogue in this country many years ago. On the regular Tontine plan the failure to pay a premium worked an absolute forfeiture of all rights under the policy, no cash or paid-up insurance being allowed. If the companies using this plan had been economically managed large profits would have resulted to the fortunate persons who continued their policies to the expiration of the dividend term. The system which has been in use for 20 years past—known as the Semi-Tontine Plan—applies the Tontine idea to dividends only, and so presents a much smaller basis for profits from the discontinuance of policies. It now appears to be losing favor and may not be used much longer for new policies.

There are excellent arguments in favor of both systems of dividend distribution, annual and deferred. Annual dividends are specially suited to a man who wishes to keep down his insurance expenses to the least figure possible. They also tend to make a company economical in the transaction of its business, because any extravagance in management will be immediately exposed by the resultant reduction in the funds available for the annual dividends. On the other hand, there is danger under the annual dividend system, that competition may lead a company to distribute too large sums in dividends, and thus risk becoming insolvent.

Under the deferred dividend system the full premium must be paid each year, and there is added to the policy an element of investment; but even if death occurs during the period when no dividends are allowed, there is still in most cases a very satisfactory return for the premiums that have been paid.

From the standpoint of a company's solvency the deferred dividend system is more advantageous than the annual dividend system, because the fact that dividends are deferred allows the company to hold large sums subject to no definite liability, which could thus serve as an extra resource in time of financial crisis, or business depression, when the company's assets might suffer extreme depreciation. For similar reasons it is the preferable sys-

tem of dividend distribution for a new company with comparatively small business and assets, and whose success is not yet fully assured. The great fault connected with the deferred dividend system heretofore has been that the companies have allowed their agents to use more or less extravagant estimates of the amount of dividends that would be realized by persistent policy-holders. These estimates were not guaranteed, but were frequently presented in such a way as to lead the policy-holder to believe them so. The fact that these estimates of future dividends have in almost all cases proved to be far greater than the dividends which finally were paid has led policy-holders to feel that they have been grossly deceived by many of the companies using this system.

Recent investigations have shown that the large unassigned surplus funds, usually connected with the deferred dividend system, have generally proved an irresistible temptation to extravagance, as no accounting was necessary until at the close of a long period, and but little positive evidence of such wasteful management would come to the knowledge of the policy-holders until too late to restrain the companies' officers. This fact, together with the disappointing dividends received by so many thousand policy-holders, has led to a tremendous outcry against the deferred dividend system, and to an attempt to prove that the whole idea of dividend accumulation is morally wrong, depriving those who die or lapse their policies of their fair share of the profits of the companies. As has been said before, however, the accumulation and possible forfeiture of dividends is a clear matter of contract, and has been in many cases the point which decided the insured to take policies; so that argument will not hold. The utter failure of the companies to fulfil expectations is, however, just ground for complaint, and there is little doubt that, if the dividends realized had been as great as people had been led to hope, we would never have heard of the present movement to abolish the deferred dividend system. There is nothing wrong in the principle of the system. If a man prefers to forego dividends for a time and thus pay rather more for his insurance, regarding it partly as a long-term investment, the deferred dividend policy may be the best for him. But he must remember that what he is taking is partly an investment, and like any other investment may not prove as satisfactory as he had hoped.



On the other hand there should be no deferring of all accounting to policy-holders until the end of the dividend term, for that tempts the companies to indulge in extravagant expenses. At least as early as at the end of the fifth policy year, and annually thereafter, a company operating on the deferred dividend plan should state to each policy-holder the share of surplus standing to his credit and contingent upon the continuance of his policy. Such exhibits would enable each man to judge whether the company was doing as well for him as other companies were doing for his friends insured in them. It should be stated that some excellent companies, employing the deferred dividend system, have always made an annual accounting by which policy-holders could judge as to their management.

Where dividends are to be made annually, the situation is quite different. Here the agent says, in effect: "My company has insurance for sale—something which your reason or your conscience will tell you that you should have—and this is the price it charges. If the company finds that the cost of maintaining your insurance is less than what you have paid,—and there is good reason to expect this,—you will be entitled to a yearly return of the excess. This you can use toward paying further premiums; or, if you feel the need of more protection, you may with it purchase additional insurance." There is nothing of investment in this proposition. A certain maximum amount may be spent each year for insurance protection. If the insured takes his dividends in cash, his insurance remains the same as before and he has in his own control the spending or investment of the savings. Some men feel that the annual dividends would indeed be spent, and be of no particular advantage, so that they might as well be left with the company to accumulate as an investment. Others are not attracted by this type of investment, and wish to spend as little as possible for their insurance, which they regard as an expensive necessity.

With the non-participating policy the idea of insurance, as an article for sale by the company, is still more strongly marked. Here the insured is liable each year for a definite, unchanging sum, less than he would pay for a participating policy. The



company tacitly agrees to stand the loss if the insurance should cost more than he has paid, but in return for this risk of its capital it takes as profit whatever it may save by carrying on its business for less money than it receives from policy-holders.

Each type of policy has characteristics which will be more, or less, attractive to different personalities.

**ACCELERATIVE ENDOWMENT PLAN.**—In closing the subject of dividends it may be well to speak of a system in use by some companies operating on the annual dividend plan, by which the dividends on a policy are used to hasten its maturity. When dividends are applied in this manner, a life policy may be made to mature as an endowment at some advanced age, or an endowment policy may be brought to maturity some years before the date originally set. The basic idea of this plan is that the dividends, instead of being used to reduce the premium payments or to buy additional paid-up insurance, are applied as single premiums to purchase pure endowments, which shall so supplement the regular reserve for the policy that, at the close of some future policy year, prior to the insured's death or to the normal time of maturity, the reserve for the original policy and the pure endowments taken together shall equal or exceed the face amount of the policy. When this happens the policy may be surrendered for the aggregate amount. Just how favorable a result will be obtained in this manner depends on the size of the dividends allowed, and these cannot of course be guaranteed. With fairly large dividends, however, a life policy issued to a young man may be made payable as an endowment at the age of sixty-five or seventy, or a twenty-year endowment policy may be brought to maturity from two to four years earlier than originally contemplated.

## CHAPTER XV.

## GOVERNMENTAL SUPERVISION.

OWING to its peculiar features, the business of life insurance is one in which some degree of governmental supervision is specially needed. In most other kinds of business the usual individual transactions can be understood by a man of average intelligence, and do not ordinarily extend for many years in the future. The contracts now issued by life insurance companies, however, generally extend over long periods of time, and are not easily comprehended by people of only ordinary capacity. A life policy on a young man may run seventy or eighty years before ending by his death: in fact it is conceivable that the special provisions in some of the policies now issued may not be fully performed until the expiration of much more than a century from the time of issue! As this may seem hardly credible, it may be well to show how this could happen: for example, suppose a man aged 25 takes a policy which gives the beneficiary a "continuous instalment" option; if he dies 40 years after and his only heir is a child 5 years old for whom the continuous instalment option is elected, the annual payments might continue for eighty or ninety years after the father's death, or until 120 or 130 years from the date of the policy.

While the ordinary man may feel that he has sufficient general understanding of the details of most other kinds of business to satisfy himself by his own investigation as to the solvency and reliability of the concerns with which he is doing business, he does not have the same confidence in his own inexpert opinion regarding a life insurance company's stability. He knows in a general way that the insurance company must have in its possession large sums of money, collected from a great number of individuals in all parts of this country, and perhaps foreign countries as well. He knows that his contract with the insurance company may remain unfulfilled for a long time and that there are many others of the same sort. He knows that immense sums must be cared for by the company, and carefully invested, and he knows the

dangers to which such funds are subject. He has, however, at the best, only a very hazy idea of how the business is carried on. To a certain extent these remarks also apply to other kinds of insurance.

For the above reasons the State governments in the United States have undertaken a general supervision of the entire insurance business, and as in many instances a single company has contracts of insurance outstanding in all the States of the Union, it has been strongly urged that the Federal government should supervise the business, under its power to regulate inter-state commerce. According to the highest authorities, however, it appears that a change in the Federal Constitution would be necessary to enable Congress to legislate in this respect, and in the absence of national supervision the entire duty of oversight of the business falls on the several State governments.

Under present conditions, a life insurance company incorporates under the laws of a particular State, and after being allowed to do business there, may extend its operations into other States so far as it obtains permission from them. All of the more important States have insurance officials, appointive or elective, whose duty it is to enforce the laws of the State relating to insurance, both as to domestic companies and as to those of other States. In this way a company that does a large business may be under the supervision, directly or indirectly, of a great number of separate State governments. Some States have quite a body of statute law in connection with insurance, and others have very undeveloped legislation. Reliance is placed on the State supervisory officials to see that the companies, both those of their own States and of other States, are solvent and also managed honestly and in such a way as not to endanger their future solvency. To this end they are given power to make such examinations of the affairs of the companies as they deem necessary to determine these points.

**ANNUAL STATEMENTS:**—Each State requires from every company doing business within its borders an annual statement as to its operations and condition. In the early months of each calendar year the company must submit to the State authorities a report, according to a prescribed form, showing in detail its assets and liabilities on Dec. 31st of the calendar year just past, and also the source and disposition of all funds received and paid out during

that year. This report is in the general form of a balance sheet, and is practically uniform throughout the United States. In it the company shows the assets on hand at the close of the previous year increased by the premiums and income on investments during the year, and decreased by the death claims paid, endowments matured, cash surrender values given, dividends allowed, and the outlay for expenses, all in considerable detail. The funds remaining, together with certain items, which, though not actually collected, are considered certain of collection, constitute a company's assets on Dec. 31st. As an offset to this are the company's liabilities, which consist principally of the reserves for outstanding policies on that date, any unpaid death claims, and the capital stock of the company, if any, the balance being surplus. The total reserves on policies outstanding is an insurance company's principal item of liability and requires explanation.

RESERVES ON DEC. 31ST YEARLY:—We saw in Chapter IV that if a company is to be sure to meet all its obligations, it must hold a reserve on each policy at all periods during the policy's existence until it matures or terminates. This moral obligation has been made a legal obligation on all companies which guarantee a fixed amount of insurance in return for a fixed yearly premium. The minimum reserve with which a company must charge itself as a liability varies in the different States. The principal States require that the reserves held on policies issued since the beginning of 1900 or 1901 shall not be less than would result if a company's policies were based on the American Experience Table with  $3\frac{1}{2}$  per cent. interest. Some States are satisfied with reserves on the same mortality table with 4 per cent. interest. Some require that if a company bases its premiums and surrender values on a lower rate of interest than that assumed as a standard, it must hold the higher reserves resulting from such an assumption. As for policies issued prior to 1900 or 1901 most States allow, as a minimum reserve, that based on the Actuaries Table with 4 per cent. interest. If it were to become apparent that the companies having such policies could not earn 4 per cent. on their assets it might be that they would be required to hold higher reserves.

The reserve held on its policies by a company on Dec. 31st of any year is not, however, the terminal reserve described in



the previous chapters. The terminal reserve may be defined as the amount to be held at the close of the particular year of a policy's existence, which may or may not be at the close of the calendar year. If a policy were issued and dated the first or second of January, the end of its first and every following policy year might then be considered to fall on Dec. 31st and coincide with the close of each calendar year. For that policy, therefore, the proper reserve on Dec. 31st could be considered to be the terminal reserve already described. As a matter of fact, however, comparatively few policies are issued with such dates, so that the proper reserve to be held for most policies on Dec. 31st must be something different from the terminal reserve.

At the beginning of each policy year, independently of the calendar year, the company should have on hand the net premium then paid and whatever terminal reserve had existed on account of the policy at the close of the previous policy year. These two sums taken together constitute the "initial reserve" for the policy year just begun. During the new policy year this initial reserve, as illustrated in Chapter IV, is increased by interest and decreased by the fact that it must be drawn upon to pay death claims as they occur. Accordingly, then, as the interest earned is greater or less than the policy's proportional share of the death claims, the terminal reserve will be greater or less than the preceding initial reserve. A nearly exact valuation for Dec. 31st of a policy issued at some other date than January 1st can be obtained by adding to, or subtracting from, the initial reserve such a proportion of the total increase or decrease, respectively, in the reserves for the current policy year, as the exact time elapsed from the policy's anniversary to Dec. 31st bears to the whole year.

This mode of valuation involves a great deal of labor and does not repay the effort, for an approximation to the exact reserve will serve every practical purpose quite as well—there being no payment of money directly involved. For this reason that method of exact valuation was long ago given up, and for many years valuations were made according to the month in which the policy was issued, each policy being valued as though issued on the 15th day of the month of its issue.

Now, however, a still more simple system has been adopted and it is customary to hold on Dec. 31st what is known as a "mean reserve," or a "mid-year reserve." It is assumed that

the policies are issued in about the same amount each business day of the calendar year, so that we may consider that, on the average, the issue date of all policies is July 1st, the middle of the year, and hold reserves for all policies on Dec. 31st, as though they were on that date an exact number of years and half-years old. That is, the policies issued in the calendar year just ended are all considered as one-half year old; those issued in the previous year one year and one-half old, those issued in the next previous year, two years and one-half old, and so on for each previous year's issue. This system allows us to hold on Dec. 31st for each policy the initial reserve for that policy increased by half the total increase in reserve, if any, during its policy year, or otherwise decreased by one-half the corresponding total decrease in reserve for its policy year. The same result may be found by taking the mean of the initial and terminal reserves of the current policy year; that is, adding them together and taking one-half the sum. This present method of valuation saves an immense amount of labor and gives satisfactory results.

In illustration of the above we take the case of a policy issued April 1st, 1902, with a net annual premium of \$20. On April 1st, 1906, the policy would have been exactly four years in force, and let us suppose it then has a terminal reserve of \$49. On that date the \$20 premium then paid would, with the \$49 of *terminal* reserve, make up the \$69 of *initial* reserve for the fifth policy year beginning that day. April 1st, 1907, would mark the close of the fifth policy year, when we will assume that the policy would have a terminal reserve of \$63. The reserve is thus \$6 less at the close of the fifth year than at the beginning. On Dec. 31st, 1906, the policy would be exactly  $4\frac{1}{2}$  years old. Its exact reserve would therefore be \$69 less  $\frac{1}{4}$  of \$6, i. e., \$69 — \$4.50, or \$64.50, on that date. If another policy had been issued on Oct. 1st, 1902, similar in all respects to the first one, its *initial* reserve for its fifth policy year beginning on Oct. 1st, 1906, would have been \$69, and its terminal reserve on Oct. 1st, 1907, would be \$63. On Dec. 31st, 1906, this latter policy would be  $4\frac{1}{4}$  years old, and its exact reserve on that date would be \$69 less  $\frac{1}{4}$  of \$6, i. e., \$69 — \$1.50, or \$67.50. The combined reserve Dec. 31st, 1906, on the two policies would then be \$64.50 + \$67.50 = \$132. On the principle that policies are issued in nearly equal amounts throughout the year, we would assume that both of these

policies were issued July 1st, 1902. The beginning and the end of their fifth policy year would thus fall on July 1st, 1906, and July 1st, 1907, respectively, the middle of the policy year being Dec. 31st, 1906. The reserve for each policy on that date, according to this assumption, would be \$69 less  $\frac{1}{2}$  of \$6, i. e. \$69 — \$3, = \$66. (Or, to get the same result, take one-half of the sum of \$69 and \$63). Then the reserves Dec. 31st, 1906, for the policies would be twice \$66, or \$132, which is the same result as obtained in the more exact manner.

(If the reader wishes to make up some exact figures for himself he will find that this assumed case corresponds very closely to the actual figures for an ordinary life policy for \$1,000 issued at age 35 when the premium and reserves are based on the American Table with  $3\frac{1}{2}$  per cent. interest.)

As payment of premiums annually in advance is assumed in all valuations, the companies are allowed to take credit for the portion of the current policy year's premiums, i. e., deferred premiums, which will fall due after Dec. 31st, less the "loadings" in those premiums. Thus, if a policy was dated Sept. 1st, with premiums of, say, \$7 payable quarterly, there would be two quarters outstanding on Dec. 31st, viz.: those for March 1st and June 1st, amounting to \$14, and if the "loading" in this sum was \$3, the company would be allowed to take credit for \$11 as an asset.

ADDITIONAL SCHEDULES:—Besides the particulars in the balance-sheet, the company must give a statement showing the number of policies, and amount of insurance in force at the beginning of the year in question, the amount of "new business," (i. e., policies taken out during the year), and the amount of insurance terminated in various ways during the year, the remainder being the total insurance in force on December 31st.

Schedules are also required to be given by the company showing in detail the various kinds and amounts of assets it is holding to meet its liabilities. Thus, its real estate mortgage investments are listed in such a way as to be easily identified, and its bonds and stocks are detailed in the same way, with the company's estimate of the market value in each case. The loans on collateral, that is, the loans secured by bonds or stocks, are also reported in detail, and a very complete schedule of real estate holdings must also be furnished.



**GAIN AND LOSS EXHIBIT:**—In addition to these schedules, the companies are required in most States to make up each year, a Gain and Loss Exhibit for the preceding calendar year. This is arranged to show just how much the surplus of the company at the beginning of the year was increased during the year by low mortality, by interest in excess of what was necessary to maintain reserves, by profits from the sale of securities or other assets, by profits on discontinued policies, and through expenses being less than the loadings in premiums received; it also shows what offsets there may have been to the above increments in the way of dividends to policy-holders or to stockholders and the cases where losses instead of gains have resulted in the items where profit ordinarily occurs. This information, when tabulated for each company, is intended to serve as a basis for comparison between companies as to the relative economy and ability with which their business is carried on. Further and fuller reference to the Gain and Loss Exhibit is made in Chapter XVIII.

**EXAMINATIONS:**—The laws also direct the State officers to make examinations of the affairs of the companies periodically or when occasion seems to demand it. At such a time the assets of a company are actually inspected and appraised and its liabilities determined; and the cash-books, ledgers and policy-registers together with the minutes of the board of trustees, or directors, are examined; in this way the tenor and condition of a company's business is ascertained and wrong-doing or weakness is surely detected.

If the State officers are honest and efficient, there is thus a fair degree of certainty that anything in a company which endangers its solvency will be found and corrected before it can do much harm. It is generally conceded by experts that a company may still be a good way from actual insolvency even when technically insolvent under the strict rules laid down by law and official discretion. If a company does become technically insolvent, that is, if it fail to have on hand the full reserve required by law, it can usually arrange to have some strong company take over its assets and become responsible for its contracts. Such an arrangement is called a reinsurance of the first-named company by the second, and in most cases it involves little, if any, loss to the policy-holders in the insolvent company.



One of the dangers arising under strict governmental supervision, based on somewhat arbitrary standards, is that there is a tendency to regard the State as guaranteeing the solvency and economical management of companies. It is forgotten that the best laws are worse than useless if not enforced, or if administered with partiality. The present disposition to enact laws calling upon the companies to publish more of the details of their business will tend to relieve the public from having to rely entirely upon the supervision by State officials. Though it is not to be supposed that people generally will give much attention to the additional information, we may be sure that competing companies will take care that their agents shall learn of the faults thus disclosed in other companies.

INVESTMENTS:—In order that policy-holders may be protected as far as possible against losses by unwise or speculative investment of their funds, the companies are restricted quite closely in the matter of investments. There are five kinds of investments open to insurance companies. They are (1) real estate mortgages, (2) bonds, (3) stocks, (4) loans on collateral security, (5) real estate.

The first class, real estate mortgages, bring a high average interest return, combined with excellent security, if due care is exercised. When loans are made in moderate amounts, and for short terms, as first liens on improved property, where there is a fair margin of valuation in excess of the loan (It is usually fifty per cent.), the risk of ultimate loss of principal or interest is extremely slight. To judge whether a company's mortgage loans are well placed, examine its annual statement and find how much interest on mortgages is due and unpaid; if it is small in proportion to the mortgage interest actually received during the year, it is evident that on the average the company's mortgages are quite good. In this connection it should be noted that State reports sometimes wrongly lump together in one item the overdue interest and the interest that is accrued but not yet due. When this is done the public cannot determine how much of the total is "overdue interest;" the actual report of the company to the insurance commissioner does state the facts, however, and he will communicate them to any inquiring policy-holder.

The companies are allowed to invest in the bonds of the United States, and individual states, and of counties, cities, etc., if there seems no reason to doubt their security. Bonds of railroads and

industrial corporations are also allowed, provided the security is excellent. This class of investments, when carefully chosen, afford security and are readily converted into money. The demand for such securities, however, is now so great that only a moderate interest return can thus be realized.

Stocks have been allowed as investments, by the laws of many of the States, subject to about the same rules as apply to bonds. In many cases the security of such stocks as the companies are permitted to hold as assets is nearly, if not quite, as great as in the case of bonds, and the interest yield is generally somewhat higher. There is also a ready market for their sale when necessary: Stocks, however, are subject to violent fluctuation on quotation, and it is, for this reason, sometimes difficult to assign to them a value accurately. There is also danger that money may be lost to a company through the speculative purchase and sale of stocks, as interests other than those of the policy-holders might be permitted to influence such investments, or that the funds of the insurance company might be used to control the management of the other company whose stock is held. For these reasons the charters or by-laws of some companies forbid investments in stocks, and now the laws of some of the States forbid such investment by domestic companies and require the sale of the stocks now held by those companies.

The fourth type of investment, loans on collateral, may be divided into two classes, viz.:—policy loans, and loans where stocks and bonds are the security. Loans to policy-holders, with their policies as security, rank with real estate mortgages as to high interest return, and even better as to security. Besides, they are often a means of holding in force policies which otherwise would be allowed to lapse. Generally a company is bound to allow a loan to stand so long as the reserve on the policy furnishes security, and for this reason the loan cannot be called in at will. The other class of investment, where stocks and bonds, such as might be bought by the company, are pledged to it to secure a loan “on call,”—i.e., from day to day,—or for a short period, affords the company, with slight risk, a convenient means of employing its funds while waiting for an opportunity to make a desirable permanent investment. It also allows a fair interest return on funds which the directors desire to keep free for use in emergencies calling suddenly for large amounts of cash.

Real estate is known to be a somewhat dubious mode of investment. It may be extremely profitable in some cases and may cause considerable loss, either of interest or principal, in other cases. It is also very difficult in many cases to obtain a satisfactory valuation of real estate owing to the fact that there is seldom a ready market for it, and little basis for comparison with contiguous property. For these reasons an insurance company is allowed to hold only such real estate as is reasonably necessary to the transaction of its business, and such as it may have acquired under foreclosure proceedings, and be holding until it can obtain a satisfactory price.

There are, therefore, really only four ways in which a company is free to make investments. The total of policy loans depends solely on the demand. The volume of mortgage loans must not be so great in amount as to put the company in danger of being short of ready cash, and it must hold a certain amount of the more easily convertible forms of investment even though the interest earned thereon is less.

## CHAPTER XVI.

## COMPANY MANAGEMENT.

APART from the peculiar features connected with the departments under the direction of its Actuary and its Medical Examiner, the internal management of a life insurance company is much the same as that of any other corporation doing a business which involves the collection, investment, and disbursement of large sums of money. It may be well, however, to mention some of the principal points in a company's organization and business methods.

ORGANIZATION:—A company's Board of Directors or Trustees has general control and supervision of all the affairs of the company. It appoints the principal officers, and directs the general business policy of the company. Various standing committees of directors are usually formed to act for the whole Board in the supervision of particular departments of the business. The duties of the several committees may differ in the attention required of their members, and they meet as often as necessary for the convenient transaction of business. At such meetings they receive reports from the heads of particular departments and consult with them as to future action. The entire Board meets once a month, or less often, to receive reports from its committees and the principal officers, and take such action on them as may seem desirable. The Board of Directors, however, usually delegates the executive control of the company and all matters of detail to its President and the other officers associated with him.

On the President of the company, therefore, lies the responsibility for the efficient transaction, by those under him, of all branches of the company's business. He should be well acquainted with financial matters and thoroughly experienced in some, or most of the departments of life insurance, so that his decisions and recommendations may be made understandingly, and thus be for the best interests of the company. Companies usually have one or more vice-presidents, who often also hold other official positions, such as Actuary or Secretary, and thus have the



special duties connected with those offices, besides those shared with the President.

The Treasurer is responsible for the oversight and safekeeping of the company's investments, and the prompt collection of all moneys due the company thereon. It may also be his duty to select new investments, subject to the approval of the President and a committee of the Directors. He may also be in general charge of the company's bookkeeping.

The Secretary has general charge of the company's records and correspondence, the preparation and issue of its policies, and often performs many important executive functions.

The Actuary has charge of all matters directly connected with the scientific basis of the business; he directs the compilation of the regular premium tables and of all the special rates required from time to time, also the preparation of tables of loans and surrender values, paid-up policies and extended insurance, also the yearly calculation of the reserves to be held for policies. He advises regarding the amount of surplus to be divided and directs the detailed allotments to individual policy-holders. Very many special calculations for peculiar cases have to be made from time to time in his department and under his general direction. As he must necessarily have an exact understanding of all the points connected with the policy contracts, so as to prepare premiums exactly suited to them, the drafting of the policy contracts is frequently one of his special duties. He also advises regarding the expenses that can be borne by the premiums, and as the science and the practice of the business are intimately connected, it thus becomes necessary for him to understand all branches of the business.

It is the Medical Director's duty to examine into the qualifications of the physicians who are proposed to serve as medical examiners for the company, and to guide in the performance of their duties those whom he selects to act in this capacity. He also gives his decision upon such applications for insurance as are submitted to the company after approval by the local examiners, and advises the officers on all matters where special medical knowledge is required.

A company's agents, scattered over a large territory, are usually put under the general control of one of its principal officers at the "Home Office," who is aided by a Superintendent of Agents, and

such other assistants as the size of the business demands. Agents are in most cases paid either by a "brokerage," i. e., a certain percentage of the first year's premium on each policy issued through their efforts, or by a somewhat smaller percentage commission on the first year's premium, and a still smaller percentage on the second and subsequent years' premiums, as collected. Thus the latter mode of remuneration may be a 40 per cent. "first year's commission," to be followed by several years' "renewal commissions" of 5 per cent. on the actual collections when made. In some cases soliciting agents are paid by salary, but this method is seldom used when it can be avoided.

APPLICATIONS:—The application form prescribes certain preliminary questions to be asked by the agent so as to clearly identify the applicant and state his age, residence, occupation, and habits, and also give full details as to the amount and kind of policy applied for, together with a statement of all life insurance policies already on his life, and the applicant's own opinion as to his health. He is also required to subscribe to certain stipulations as to the issue of the policy, and often also as to limitation of risk under certain conditions. Frequently the first premium is paid at the time of making application and is received subject to the acceptance of the application at the Home Office.

If there appears to be no objectionable feature in the application, a medical examination follows as soon as practicable. The physician begins by asking a series of questions as to the physical history of the applicant's parents and family. These queries are calculated to elicit all the facts, whether favorable or unfavorable; those tending to show an inherited tendency to longevity on the one hand, or a liability to some hereditary or constitutional weakness on the other hand. Questions are asked as to his own past sicknesses and accidents, and also as to the extent of the applicant's past and present use of alcoholic stimulants, or narcotics.

When the applicant has signed his name to the answers given, the physician proceeds to determine, by a personal examination of the condition of the heart, lungs, and other organs, as well as by exterior evidences of physical condition, weight, height, appearance, etc., whether or not the person proposed for insurance is an acceptable risk. He then makes a written confidential report on these matters, stating his own conclusions, and forwards it immediately to the Home Office.

**INSPECTION:**—The company always requires the agent soliciting an application to certify to his belief that the risk proposed is a desirable one in all respects. For small amounts of insurance this certificate and the application are in most cases regarded as sufficient data on which to pass final judgment. Otherwise an "inspection" report is ordered. Then, through a special detective service, independent information is obtained as to the applicant's reputation, habits, financial standing, evidences, if any, of tendency to insanity, and anything else about which further independent information seems desirable. It should be remarked, however, that this investigation is not accompanied by any circumstances that could be objectionable to any man with a good record, and is only such as the company ought to make before assuming a risk of many thousands of dollars, when it is remembered how many fraudulent applications have been discovered in the past.

**ACCEPTANCE OR REJECTION:**—From this set of data regarding the applicant decision is then made, by the officers at the "Home Office," whether, or on what conditions, the risk will be accepted. The company has first to determine whether the vitality and environing conditions of the applicant are up to the standard for longevity, and second to guard itself so far as possible against what is known as the "moral hazard."

The mortality tables suppose all persons to be in good health at the time of entering a company, and the premiums are based on this assumption. Therefore, if any considerable number of persons who are not of this quality were allowed to come into a company the mortality experienced would be greater than that assumed in the company's calculations, and would cause embarrassment, if not failure. The duty of deciding this question of whether a life is up to the standard or not rests largely with the Medical Director, the "inspection" report serving, as the case may be, to verify or throw suspicion upon answers made at the medical examination. This report is also intended to guard the company against accepting risks on insane persons or those who have shown a tendency to insanity, for such persons are liable to commit suicide. When a large amount of insurance is sought it is also necessary to find out the applicant's financial standing, for if a man applies for a greater amount of insurance than he can readily pay the premiums upon, there is good ground for suspicion



that suicide, or fraud of some kind, is contemplated. It is true that clauses in the policy may limit the amount payable in case of suicide, sane or insane, in the first one or two years of a policy, or render the policy entirely void in such case; but they do not afford complete protection against the payment of suicide claims, for in court it is often difficult or impossible to prove the fact of suicide to the satisfaction of a jury. Lastly, the report is intended to detect any fraud against the company by collusion on the part of the examiner, or by the substitution, for the purposes of the examination, of a healthy man in place of a sickly one. If everything is satisfactory the policy is issued, generally being given to the agent for delivery.

**REINSURANCE:**—Ordinarily a company sets a certain limit to the amount of insurance it will carry on a single life. This is done to prevent the company being suddenly called on to pay out, unaided, a very large sum on a single death claim, or a still greater sum if two or more such large policies should happen to mature at about the same time. In order to avoid having to refuse to accept an application for an amount of insurance greater than this limiting amount, a company will accept the application, and “reinsure” the amount in excess in some other solvent company. This means that it will take out in the other company a policy, payable to itself, on the same life, for the excess amount. Then if death occurs the original company will pay the whole sum insured, but receive a portion thereof from the company in which it placed the reinsurance.

**“SELECTION:”**—From a very large experience it has been found that the mortality actually met by the companies on lives newly examined is very much less than on lives of the same age which were examined some years earlier. Such newly examined persons are not less liable than others to death by accident; but, owing to their general good health at the time of admission into the companies, there are few deaths from disease; thus this peculiarity of what are called “select lives” is explained. It is also found that the effect of this “selection” becomes less and less pronounced as time passes, and almost entirely disappears in the course of the first five years after date of examination.

This fact of “selection” has long been considered as established, among insurance companies, and the funds which can be saved out of premiums, charged as though there were no such selection,



are counted on as a basis for dividends or to meet the expenses necessarily connected with the issue of a policy. This "selection" will often explain the very low percentage of the "actual" to the "expected" death losses in a company newly organized, or in one which has recently grown with great rapidity, for in such cases a large majority of the persons insured will have been recently passed upon by the medical examiners.

**STANDARD LIVES:**—Many persons who are not fully acceptable to a company may yet obtain insurance, but only on conditions which vary according to the degree of the impairment, or the lack of entire acceptability, of each life in question. Where this is very slight, the applicant may be accepted for Endowment insurance for twenty years or less, because by this arrangement the company receives a certain protection in the rapidly increasing reserve, which diminishes the amount at risk, and also from the fact that all risk terminates with the endowment period. If the life is still more "under-average" a premium may be charged for an age higher than that of the insured, or a policy may be issued at regular rates, but with a lien standing against it and arranged to slowly decrease as time passes, until finally it is cancelled.

**EXTRA HAZARDS:**—When the insured's occupation or residence renders him specially liable to accidents, or is specially injurious to health, the risk, if accepted at all, will usually be taken only subject to an extra premium.

**INSURANCE ON WOMEN:**—Most companies will, under certain conditions, issue policies on the lives of women, but their practice in this respect differs considerably. The general rule is that insurance will be given only where the woman is self-supporting, so that there may be a *bona fide* insurable interest in her life.

**EXPENSES:**—The expenses which an active insurance company must incur may be roughly divided into two classes:—"general expenses," and "expenses connected with the issue of policies." "General expenses" comprise the commissions on renewal premiums and the greater part of the outlay for salaries, rental, taxes, care of investments, and the like. "Expenses connected with the issue of policies" or "first-year expenses," as they are sometimes called, include the large commissions or brokerages on initial premiums, medical examination and inspection fees, most advertising expenses and unsecured advances to agents. The

latter class, first-year expenses, is always much greater than the former in proportion to the amount of premiums received in each case. In many instances all of the initial premium is used up to meet the many expenses connected with the issue of the policy. The general experience in the past has been that the excess of first-year's expenses for policies is not fully repaid by the margins in the premiums until the policies have been in force for about five years.

This state of affairs has proved a great hardship to new and small companies having but little surplus to spend in obtaining business. In an old company the margins in renewal premiums are more than enough to cover "general expenses," and can be used to supplement the margins on the first premiums on new policies in payment of initial expenses. By this means, or by using some of its surplus accumulated from old policies, an old company may be able to provide the full legal reserve for new policies, though it could not be saved out of their own premiums after paying the heavy expenses.

In a new company, however, the amount of renewal premiums is small, so that what have been classed as the general expenses, as well as the initial expenses, fall almost entirely on the new premiums. Only the possession of a large surplus fund will allow a young company to hold the full legal reserve under those circumstances—without impairing its capital. Therefore, nearly all the new companies have been forced to adopt what is called the "preliminary-term" system of business.

"PRELIMINARY-TERM" RESERVES:—If a policy is issued reading in substance that "the payment of the first premium will give insurance for the term of one year, but the policy will be continued thereafter as a whole life policy on the payment of the same premium at the close of the one-year term and annually thereafter during life," the insurance given is the same as it would be under a simple whole life insurance policy. Technically, however, the above language, taken with a corresponding provision in the application, makes the contract consist of two parts, viz.:—a one-year term insurance, combined with a whole life insurance beginning a year later at an age one year greater.

Take for illustration a regular whole life policy for \$1,000 issued at age 40 with a gross annual premium of \$32. The net premium in this case, on American  $3\frac{1}{2}$  per cent., is \$23.50, so that the margin in each year's premium is \$8.50.

The corresponding "preliminary-term" policy may be issued for the same gross premium of \$32. The net premium for the first year's term insurance is \$9.46, and then the net premium for each year thereafter is \$24.36, which is the net premium for ordinary whole life insurance at age 41. The margin in the first gross premium would therefore be  $\$32 - \$9.46 = \$22.54$ ; in each subsequent premium it would be  $\$32 - \$24.36 = \$7.64$ .

The reserves which must be held for each policy by legal requirement on December 31st of certain policy-years compare as follows:—

**\$1,000 WHOLE LIFE INSURANCE, AGE 40, AMERICAN THREE AND ONE-HALF PER CENT. MEAN RESERVE ON DECEMBER 31.**

Policy Year.	1	2	3	5	10	15	20	30
Regular Whole Life Policy.....	19.09	33.99	49.35	81.43	169.34	266.32	368.97	573.24
" Preliminary-Term " Policy.....	4.73	19.86	35.45	68.00	157.22	255.65	359.82	567.14
The former exceeds the latter by.....	14.36	14.13	13.90	13.43	12.12	10.67	9.15	6.10

The first line gives the regular "mean reserves" previously described. In the second line the first-year's mean reserve is simply one-half of the one-year term premium at age 40, as the terminal reserve is 0. The second and subsequent years' mean reserves are the regular mean reserves for whole life insurance beginning a year later at age 41.

At the issue of the "regular" policy the margin of premium available for expenses in connection with the policy's issue is \$8.50. With the "preliminary-term" policy the corresponding amount is \$22.54, or \$14.04 more. In each case a saving from mortality can also be counted on to meet expenses. At the close of the calendar year the difference in reserve is \$14.36. Accepting it as a general rule, therefore, that the whole gross premium of \$32 must be used up to cover insurance and expenses in the first year, the company operating on the latter basis has about \$14 more to use in initial expenses without assigning its surplus funds to the duty of acting as a reserve for the insurance.



Of the \$8.50 margin in the second premium on the former basis something must go to meet general expenses. The remainder can go toward reimbursing the surplus fund for the sum previously used to furnish a first-year reserve. In the latter case the corresponding margin of \$7.64 can be used for the same purposes, but the indebtedness of this policy to the company's surplus is at least \$14 less than in the other case. The value of the preliminary-term system to a young company with a small surplus can, therefore, be readily appreciated.

In the above illustration it has been assumed that the gross premium charged is the same in both companies; but, as a matter of fact, "preliminary-term companies" can afford to charge rather lower premiums, and often do so, as their special system yields so much more for initial expenses that lower premiums will suffice.

So long as the policies of a company using the preliminary-term system clearly state the true nature of the contract, there is no deception involved. The reserve held is theoretically correct and will provide for payment of the policies at maturity. The effect is simply to make each policy issued pay the expenses of issue and not depend on other funds for this purpose.

As the reserves are smaller, the guaranteed loan and surrender values dependent on them must also be less. Generally speaking, however, a larger percentage of reserves may be allowed the retiring policy-holder, because nothing need be deducted to cover expenses previously incurred. The comparative table shows that the difference becomes less and less as time passes.

When applied to a limited payment life policy the effect of this system is to reduce by one the number of premiums which go to make the policy paid-up. Thus for a 20-payment life policy at age 40 the corresponding preliminary-term contract is a combination of a one-year term insurance at age 40 with a regular 19-payment life policy, at age 41. When applied to an endowment policy the first premium is similarly used only for expenses and the one year's insurance, and the accumulation of reserve to provide for the maturity of the endowment begins with the second premium. The regular 20-year endowment at age 40 is thus replaced by a contract combining a one-year term insurance with a 19-year endowment insurance at age 41.



While the surrender values are less for such policies on this plan, it is generally admitted by experts that those allowed by companies which operate on the ordinary reserve plan are entirely too large in most such cases, and are given only under the pressure of competition.

**"MODIFIED PRELIMINARY-TERM" VALUATION:**—The simple preliminary-term valuation system above described is somewhat open to objection on the ground of inequity between different classes of policies, and this has given rise to a modification in the system. On the simple preliminary-term plan no greater first-year reserve would be held for a 20-payment life or 20-year endowment policy than for an Ordinary whole life, though there is, of course, a great difference in the premium received. With gross premiums per \$1,000 at age 40 for Ordinary whole life of \$32, for 20-payment life of \$42, for 20-year endowment of \$52, and a first-year December 31st reserve of only \$4.73 in each case, the 20-payment life premium gives for expenses \$10 more than the Ordinary life premium, and the 20-year endowment premium gives \$20 more, though the insurance in each case is the same.

The "modified preliminary-term" plan bases the reserves on all higher premium plans upon the reserve for the Ordinary whole life policy, which is itself given the simple "preliminary-term" reserve. Thus at the close of the 20th policy year of a \$1,000 Ordinary life preliminary-term contract issued at age 40 the reserve on American  $3\frac{1}{2}$  per cent. is \$358.21, or the same as at the end of 19 years from issue at age 41. At this period, when the contract is 20 years old, the attained age would be 60 and the single premium would be \$626.92. If the policy is to be paid-up at that date, or in other words if it is to be paid for by 20 premiums only, there must be a reserve, in addition to the \$358.21, of \$626.92 — \$358.21 or \$268.71, which has to be accumulated in some way. According to the "modified" system this is done by adding to the net premium for each year of the 20 a further level net premium for a 20-year pure endowment policy for the \$268.71. The extra net premium in this case is \$7.51.

For a 20-year endowment of \$1,000 the corresponding additional reserve necessary at the close of 20 years from the policy's issue would be \$1,000 — \$358.21, or \$641.79. This is provided for by an additional 20-year pure endowment premium of \$17.94.

The effect of this system is to provide, by pure endowment accumulation running through the entire term of premium payments, for sufficient additional reserve, beyond that for Ordinary whole life insurance, to make a life policy become paid-up, or to mature it as an endowment. The gross premium, the combined net premium per \$1,000 in each case, and certain mid-year reserves, are given at age 40 on American 3½ per cent. for purposes of comparison:—

Form of Policy.	COMBINED NET PREMIUMS.		MEAN RESERVES DECEMBER 31.					Assumed Gross Premiums
	1st Year.	Other Years.	1st Year.	2d Year.	5th Year.	10th Year.	15th Year.	
Ordinary Whole Life.	9.46	24.36	4.73	19.86	68.00	157.22	255.65	32.00
20-Payment Life....	16.97	31.87	12.41	35.57	110.10	252.59	420.55	42.00
20-Year Endowment.	27.40	12.30	23.08	57.36	168.50	385.00	649.51	52.00

The result of this arrangement is to compel a company, which collects premiums on plans calling for greater premiums than on Ordinary whole life policies, to hold larger reserves, somewhat in proportion to the degree in which the premium for the higher-rate plan exceeds that for Ordinary whole life, thus reducing in the same proportion the portion of the gross premium available for initial expenses.

“SELECT AND ULTIMATE” VALUATION:—A third special system of policy valuation will now be outlined. In the year 1906 the New York Legislature prescribed the “Select and Ultimate” method of calculating reserves, as the minimum reserve standard for that State, and based upon it certain regulations limiting expenses of companies. This method of valuation differs from the usual net system only as to the reserves for the first five years, while the various preliminary-term systems affect the reserves during the entire premium-paying period.

Owing to the effect of “selection,” already described and explained, the mortality experienced by a company on a policy during its first policy year will, on the average, not exceed 50 per cent. of the “expected” by the usual mortality tables, which are called “ultimate” tables, because in their construction the effect of “selection” was eliminated and they show what the rate of mortality will *ultimately* become some years after medical examination.

In the second policy year recent selection keeps the mortality down to not over 65 per cent. of the expected by the ultimate tables; in the third year the corresponding percentage is not over 75; in the fourth year not over 85, and in the fifth year not over 95. After the fifth policy year the experience may be expected to approximate more closely to that indicated in the ultimate table, though in some companies the mortality continues lower than the tabular for very many years longer.

These savings in mortality relieve the companies just so far from paying out all the portions of the net premiums of the first five years that had been intended to pay death claims, and in practice the companies have relied on them to recoup themselves in part for the expenses connected with the issue of policies. The basic idea of the "Select and Ultimate" system is to recognize these probable future savings when determining the reserves to be held for newly issued policies. This is done by reducing the reserve, according to the full legal reserve method, by the present value of these probable mortality savings in the first five years on the conservative assumptions above indicated. The reduction of reserve thus made serves to release additional funds to meet initial expenses. After the fifth policy year the full legal reserve is held.

The computation of reserves by this method requires special tables, which may be found, together with a full description of the system, in "Practical Lessons in Actuarial Science," by Miles M. Dawson, F. A. S., the originator of the system. The resulting reserves are somewhat higher, during the first five years, than those on the Modified Preliminary-Term plan, and thereafter, as previously mentioned, are identical with the reserves by the regular net reserve system.

Below are some examples of Mean Reserves on this system which can be compared with those on the ordinary net valuation plan.

*Mean Reserves by "Select and Ultimate" System.*

Age at issue, 40; policy of \$1,000, basis American  $3\frac{1}{2}$  per cent.

Form of Policy.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.
Ordinary Life.....	\$9.59	\$28.25	\$46.32	\$63.95	\$81.15
20-Payment Life.....	17.18	43.56	69.68	95.78	121.83
20-Year Endowment.....	28.11	65.57	103.27	141.50	180.30



## CHAPTER XVII.

## INDUSTRIAL INSURANCE.

INDUSTRIAL life insurance is founded on almost exactly the same general principles as Ordinary insurance, but differs in many points of practical management. Its purpose is to provide insurance protection in small amounts with weekly premiums for the industrial classes, who are not reached and benefited by Ordinary insurance methods, partly because the people of these classes have no time or opportunity to obtain policies from—and pay premiums to—companies operating only on the Ordinary plan, and also because it would be difficult or impossible for them to set aside enough to meet larger premiums falling due annually, or even quarterly. Industrial policies for small amounts are also taken by parents on the lives of young children, to pay funeral expenses in event of death. In this way there may be, and often are, insurance policies on the lives of all the members of a family.

In Ordinary insurance the minimum policy issued is generally for \$1,000, which is the unit for premiums and surrender values. With Industrial insurance the unit is five cents of weekly premium, the insurance being such as five cents or a multiple thereof will pay for at the insured's age at *next* birthday. The policies are usually for very small amounts and either on the whole-life plan or some form of long-term endowment. The average policy is under \$150, though there are many for \$500. The application is very simple in form, and the medical examination in many cases is not expected to do much more than protect the company from accepting persons who are in obviously bad health; in fact, it is not much more than an inspection in the case of the smallest amounts of insurance.

Even with extensive organization, and the most careful attention to all details, the expenses connected with the conduct of Industrial insurance greatly exceed those for Ordinary insurance. These higher expenses are reflected in higher premiums for the corresponding amount of insurance. Thus at age 35, a five cent weekly premium will pay for \$59 insurance. Twenty-five cents



a week, or \$13 a year, will give five times as much, or \$295 Industrial insurance, on the participating basis. On the regular Ordinary plan, however, \$13 paid yearly would give about \$500 of participating insurance in some of the lower-premium companies, if they would issue policies for such a small amount.

In this connection it should be borne in mind that the cost of carrying on an Industrial insurance business compares with the corresponding cost in Ordinary insurance just about as the expenses connected with selling coal by the bucketful compare with the expenses where nothing less than a ton is sold. The large expense rate is due to the cost of making weekly collections from door to door and to the fact that immense numbers of these small policies are allowed to lapse before the comparatively large expenses, connected with the issue of the policies, have been covered by the premiums received on them. The companies do whatever seems practicable to prevent the heavy lapse rate, with its attendant expense, but have succeeded only in a measure. Policies are not considered lapsed until four weeks' premiums are overdue, and the payment of agents is so arranged that every lapse causes a direct money loss to the agent, leading him to do all in his power to keep the policy in force.

The companies doing this kind of business also find that the mortality experienced in it is very much greater than would be the case with Ordinary insurance. This excessive mortality is due partly no doubt to the less rigid medical examinations, and also to the fact that the class of persons who can pay only by weekly premiums generally do not have such wholesome surroundings as the more well-to-do, and when sick will often be unable to take proper care of themselves. Besides this, their occupations are likely to be more hazardous than those of more highly paid persons.

Industrial insurance, like Ordinary insurance, came here from England, and dates in the United States only from the year 1875. For that reason it may hardly yet have reached its full evolution here in some matters of practice. The first policies issued were without surrender values or participation in surplus. As time went on, however, it was found practicable to allow both of these, and now Industrial policies resemble Ordinary policies in these respects. Some companies' policies provide for surplus distribution at the close of a period of five, ten, or fifteen years.

Other companies issue strictly non-participating policies, but make a practice of voluntarily allowing dividends from time to time on policies which have been several years in force. The policies usually provide for paid-up insurance on lapse after premiums have been paid for three years, but cash values commonly are not allowed until after a much longer time.

Industrial insurance is rarely undertaken except by companies having capital stock. It is insurance at the smallest kind of retail and must be done on a large scale with small profits in each individual case. This involves a very considerable outlay of capital for some years, to organize a force of agents and put a fair amount of business on the books, before any return on the investment can be expected. All of the companies doing Industrial insurance carry on an Ordinary business as well. Besides the economies connected with making a double use of agencies, the companies derive advantage from the fact that those who have seen or profited by the benefits conferred by Industrial insurance are often thereby educated to such habits of thrift as to be able to bear the expense of Ordinary policies for \$1,000 or more, with premiums relatively smaller.

Several of the companies doing an Industrial business have introduced what is designated as the "Intermediate Plan," upon which they issue policies of \$250 or \$500 to persons of about the same class as are insured on the Industrial plan. These Intermediate policies have their premiums payable annually, semi-annually or quarterly, and cost appreciably less by the year than for the same insurance if paid by weekly premiums. These Intermediate rates, however, are higher than Ordinary premiums, as the death rate among those taking such small policies is rather high.

When companies first began doing Industrial insurance a great deal of educational work was necessary. People were slow to believe that the corporations collecting such trifling premiums could achieve success, and there was great difficulty in securing both agents and insured within reasonable limits of expense. One of the features of the business is that it insures the lives of young children. For this reason the Industrial companies were accused of furnishing a temptation to infanticide in any case where there might not be sufficient parental love to preclude any disposition to bring about a child's death by violence or neglect, for the sake

of the insurance money. The companies, however, have been very careful not to allow enough insurance to make the profit from such crime sufficient in comparison with the risk of detection, and laws have been passed placing limits on the amounts of insurance that can be carried upon children's lives. These maximum amounts very properly increase with the age of a child, for it is realized that after their early years children are to the industrial class rather an asset than a burden. It has been proved that Industrial insurance tends to prevent death, rather than cause it; for parents are more ready to incur a doctor's bill for a sick child, if there is an insurance policy to pay the bill in case of death.

This education of the public was hard to bring about, but persistency has done it, and now the purpose and effects of Industrial insurance are well understood in all centers of population throughout the country. Statistics show that it has greatly benefited the working classes by inculcating habits of saving, and reducing the number of pauper burials.

The popularity that Industrial insurance has attained in this country is shown by the fact that the volume of these very small policies now outstanding is almost one third as great as that of the larger policies on the Ordinary plan.

The calculations connected with Industrial insurance are made on the same general principles as in the case of Ordinary insurance, except that the premiums are not assumed to be payable annually; for this reason there are no "deferred premiums" to be deducted from a death claim, as would often be the case where Ordinary premiums had been paid otherwise than annually. Thus in the case of an Ordinary policy dated February 1st, with quarterly premiums, if the insured died in February there would be three of the quarterly instalments to be deducted from the policy. If this method were used in Industrial insurance, it would be very unpopular, as in such a case as this one just cited, it would be difficult or impossible to explain to the family of a working man why so much deduction (nearly fifty weeks' premiums in this case) should be made.

Industrial policies are valued by state insurance departments somewhat differently from Ordinary policies, but as the practice is not uniform in all the states, it is not advisable to attempt an explanation of the subject in this elementary treatise.



## CHAPTER XVIII.

## COMPETITIVE COMPARISONS BETWEEN COMPANIES.

IN the fierce competition which has arisen between the companies, each life insurance agent will bring forward as far as possible everything that is favorable to his own company, and try to show up every fault he can find, or claim to find, in a competing company. It is perfectly natural, and in keeping with the practice in other lines of business, for an insurance company to wish to "put its best foot forward," but the character of the business is such that, in some particulars, unqualified statements as to the excellence of one company and the deficiencies of a competitor may be extremely misleading and sometimes really dishonest. The purpose of this chapter will be to indicate some general criteria which will help in forming a correct judgment as to the relative merits of companies, and also to give some examples of the ways a second-rate company may be argued to be an excellent one.

**RELATIVE SIZE:**—Provided a company has in force a volume of insurance great enough to preclude any danger of embarrassment from fluctuations in the rate of mortality, the attainment of great size is not necessarily an advantage of itself and apart from other considerations. As a matter of fact, the supposed need of considerable size, in order to ensure a fair average, is really a theoretical matter, for there appears to be no record of any case in which a small company suffered from excessive death losses. A large amount of insurance in force may result simply from the fact that a company has been a long time in business. It may also be the result of very rapid increase attained at very great and even reckless cost. If a company makes little yearly increase in the amount in force, this may either be due to economical management combined with liberality toward policy-holders, or to incompetent management. Likewise rapid increase in business may result from able and economical management.

The character of the business, and the expense connected with acquiring it, are far more important points than mere size. A small amount of new insurance placed each year on carefully



selected risks and at a moderate expense, is of more value than a much larger amount obtained with less care and without regard to cost. The pernicious practice of "rebating," whereby the agent divides his commission with the insured, selling the policy at a discount as it were, is fostered when very high commissions are allowed to agents for procuring business. It is a rule in life insurance that "the business that stays is the business that pays," and it is well known that insurance placed at high expense and through "rebates," is very likely to lapse before it has paid for itself, while similar insurance issued at moderate expense and under conditions making rebating impracticable will be kept in force for many years.

Much stress is often laid on the fact that this or that company is possessed of great assets. This feature also may not be so advantageous and desirable as it seems at first. It should be remembered that ordinarily an insurance company's assets must increase, if the company is to continue solvent. Even if there were no yearly increase in a company's insurance in force, its assets must continue to increase to a relatively large amount. Its policy reserves, which constitute its chief liability, would also increase almost directly in the same proportion. An old company, or indeed any company with large assets, is therefore not necessarily stronger than another company young or old with smaller assets.

The possession of a large "surplus," i. e., the excess of assets over reserve requirements, is also held in high regard by some persons as tending to show special strength and stability. Undoubtedly this is the case, but there is such a thing as carrying this matter to an extreme. As compared with a company possessing a fairly large surplus, another company with a surplus fund five times as great in proportion, cannot reasonably be considered to be five times as secure as the first, or anything like it, for the regular reserve held by each company is the primary element of security and the surplus is only a secondary one.

This subject is closely connected with that of the distribution of dividends, for one of the arguments in favor of deferred dividends is that the great sums thus held for future distribution are available for use, if necessary, in making up any deficiency in other assets due to sudden heavy mortality or depreciation in securities.

There is great question, however, whether there is any real propriety in holding back such large sums and whether such companies would not be sufficiently safe with much smaller surpluses, while their policy-holders would be benefited by being allowed to use the redundant amount to reduce their yearly payments. In fire insurance the possession of a large surplus has repeatedly been found important, because great conflagrations have so often occurred, but in life insurance experience has never yet shown any corresponding danger.

As against the argument in favor of great size as a factor of strength, stands the fact that an immense corporation is subject to the danger of exploitation by those who are in actual, if not nominal control, for many things could go wrong in a very large company for a long time before being brought to light. It is also liable to be brought, directly or indirectly, into politics.

COMPARISON OF DIVIDENDS:—When comparing dividends, particularly annual dividends, care should be taken not to be misled by percentages. Thus a 7 per cent. dividend on a \$30 premium, or \$2.10, reduces the net payment to \$27.90. The same percentage of dividend declared on a \$28 premium would reduce the net payment to \$26.04, or \$1.86 less than in the first case. It would, however, need a dividend of only 10 cents, or about  $\frac{4}{10}$  of one per cent., on the \$28 premium, to reduce the net payment to a parity with that in the first case. Therefore, any dividend on the \$28 premium exceeding  $\frac{4}{10}$  of one per cent. will cause a greater effective reduction in net cost than a 7 per cent. dividend on the \$30 premium.

In comparing the net cost of two competitive policies during a term of say 20 years, where one was on the annual dividend and the other on the deferred dividend plan, it would not be fair to simply deduct the total dividends from the total premiums paid, for that would disregard the very appreciable interest element which should be considered in connection with annual dividends, and also some other points in which the two kinds of dividends are not entirely parallel. Though the deferred dividends on one policy may be somewhat greater than the total annual dividends on the other, even when interest has been reckoned, it should be noted that the deferred dividends would only operate to reduce the net cost provided the insured lives out the term and continues payments. The annual dividends, though less, cause actual cash

reduction of the amount of premiums paid, without the risk of their forfeiture by death or lapse.

**GAIN AND LOSS EXHIBIT PERCENTAGES:**—Supplementary to the reports which the companies must make annually to the state governments, they are required to submit itemized statements which are calculated to show the profits and losses in each particular line of their business for the past year, which would affect the amount of surplus held. Thus:—(1) The loadings in premiums received are compared with all the expenses incurred during the year for the conduct of the business except expenses in connection with investments; (2) The “expected mortality cost” is compared with the “actual;” (3) The “net investment earnings” are compared with the “interest required to maintain the reserves;” (4) The “reserves and dividends released by lapse” during the year are compared with the “surrender and lapse values allowed.” The reader will recognize that profits in these respects are sources of dividends, as (1) savings in expenses, (2) savings in mortality, (3) excess interest earnings, and (4) profit from lapses. The net gain from these sources, together with any net increase in the market values of securities (or less any net loss in this respect), is then added to the surplus existing at the close of the previous year. From this total are deducted the dividends to policy-holders and stockholders. The remainder is then the surplus for the close of the calendar year.

Ordinarily the four items of gain or loss first mentioned are used for competitive purposes, and commonly they are stated in columns of percentages, of which the following will serve as samples:—

Typical Company	Percentage of Insurance Expense to Loading in Premiums.	Percentage of net Interest Earned to Interest Required to Maintain the Reserves.	Percentage of Actual to Expected Death Loss.	Percentage of Reserves Returned on Surrenders and Lapses.
Company A.....	95	150	80	85
“ B.....	80	130	80	85
“ C.....	180	120	60	80
“ D.....	225	125	80	75
“ E.....	115	120	80	90
“ F.....	90	130	110	45



The above percentages are intended to be typical of six principal classes of companies, and it is apparent either that there is a tremendous difference in the condition of the companies or that these percentages need a great deal of explanation. The explanation is indeed necessary, and will in great measure account for the wide variation in the percentages. It should be stated, however, that no case cited corresponds exactly to any particular company now doing business.

Company "A" typifies the companies which have been in the field for many years, increasing their insurance in force rapidly and by progressively larger amounts each year for a considerable period, charging premiums with relatively great margins, and spending money very freely.

Company "B" represents another class of old companies,—those which are economical, whose premiums are relatively low, and whose yearly "new business" causes only a moderate increase each year in the total amount of insurance in force.

Company "C" stands for institutions, old or new, the great bulk of whose total insurance in force has been obtained within the last decade.

Company "D" typifies companies whose business is very largely on the non-participating plan, with small average margins in premiums, and relatively low surrender values.

Company "E" represents those transacting business on the preliminary-term plan, with its special provision for first-year expenses.

Company "F" stands for companies having a considerable portion of their business on the Industrial plan, with its peculiar features.

We will now investigate the variations in the first column of percentages. Both "A" and "B" are old companies, so that each has a relatively large renewal premium income, the expense margins in which will go far toward making up for the deficiency in the margins in premiums on new business. The average percentage of loading in "A's" premiums, however, is greater than in "B's," so that "A" could spend more money than "B" without appearing at a disadvantage in this column; yet even with this advantage "A" does actually show a higher percentage than "B," but the excess of percentage by no means indicates the real excess of expenditure by "A;" had the percentages been



reckoned upon the net premiums, the actualities would have been more nearly indicated.

"C" represents a rapidly growing company whose business is, on the average, only a few years old, so that its renewal premium income is small and the margins in its renewal premiums can contribute little or nothing toward meeting the initial expenses on the new business.

A young company must make a poor showing in this column even though its expenditure per thousand of new insurance compares favorably with that of "A." A company just started is likely to show a much higher percentage than indicated, and the percentage may be expected to decrease as the new business becomes a smaller proportion of the total insurance in force.

"D" does a largely non-participating business with premium margins perhaps two-fifths as great as those for participating policies. Such a company may, therefore, spend less money per thousand for its new insurance and yet appear to a disadvantage in this column.

"E," operating on the preliminary-term plan with gross premiums about the same as in other companies, can use for expenses a large part of the first premiums on new business; so that, though most of its business may have been recently acquired, the company may appear in this respect about as well as an old institution.

"F" is a well-established Industrial company with a relatively small amount of Ordinary business, so that on the average its premiums are loaded much higher than those of companies doing only an Ordinary business. Such a company should normally have a low percentage, as is actually shown. Such a company, however, if only recently organized may be expected to show a very much higher percentage, because of the large expenses of the early years and the absence of any considerable assistance from margins in renewal premiums.

From the foregoing remarks it should be clear that these percentages of expense afford no positive means of deciding as to the degree of economy in the management of the various companies. The old-fashioned practice of showing the percentage of expenses either to premium income or to total income gave the average policy-holder a much better means of judging as to the relative economy of the different companies.

With regard to the second column, "Percentage of Net Interest Earned to Interest Required," it will be well to make some preliminary observations. Prior to the year 1901 the reserve standard for most policies was on a 4 per cent. basis, and the companies have been permitted to continue holding reserves on that standard for all policies issued thereon. Since January 1st, 1901, the minimum reserve standard has generally been based on three and one-half per cent., and many companies, some of which had adopted a 3 per cent. standard some years before, have preferred to continue to hold the still higher reserves on that basis.

If, then, actual interest earnings average 4 per cent., a company holding reserves mostly on a 4 per cent. basis will earn about 100 per cent. of the required interest; a company reserving mostly on a  $3\frac{1}{2}$  per cent. basis earns 114 per cent. of the required; and one holding 3 per cent. reserves earns 133 per cent. If the interest earned was  $4\frac{1}{2}$  per cent., the 4 per cent. company has a percentage of 112.5 per cent.; the  $3\frac{1}{2}$  per cent. company about 129 per cent., and the 3 per cent. company 150 per cent. Similar ratios may be formed for intermediate or higher rates of interest earned.

Therefore, a company whose reserves are mostly on 4 per cent. or  $3\frac{1}{2}$  per cent. may be expected to show a less favorable percentage in this column than one whose basis is 3 per cent., though the former company may have actually realized a higher rate of interest than the latter.

A large part of "A's" business may be assumed to be on the 3 per cent., or on the  $3\frac{1}{2}$  per cent. basis. Most of "B's" reserves are probably on a 4 per cent. basis. "F's" reserves may average about the same as "B's." "C's" and "E's" reserves will be chiefly on a  $3\frac{1}{2}$  per cent. basis. "D's" average interest requirement may be about midway between those of "B" and "C."

To these assumed typical cases some notable exceptions will be found, owing to many causes that cannot be stated here. It is also important to bear in mind that the showing in this column in any one year is not to be taken as a criterion, for the figures often fluctuate widely from year to year.

In respect to these interest percentages, a further and most important point must be kept in mind, viz.: in the exhibit a company is given credit for the interest actually earned on its entire

assets, both reserves and surplus, but is charged only with the interest that theoretically should be earned on its reserves alone. Therefore, a company that has accumulated a surplus which is 20 per cent. as great as its reserves must almost necessarily make a more favorable showing than another which holds exactly similar reserves, but a surplus of only eight per cent.; yet the latter company may be obtaining a much better rate of interest than the former. To make this clear, let us suppose that both companies have reserves of \$100,000,000 on the 3 per cent. basis—that the former has a surplus of \$20,000,000 and has earned 4 per cent. on its entire assets, or \$4,800,000; also that the second company has a surplus of \$8,000,000 and has earned  $4\frac{1}{2}$  per cent., or \$4,680,000; in each case the “interest required to maintain the reserves” is \$3,000,000, and the first company will appear to be earning 60 per cent. in excess thereof, while the other will appear to be earning only 56 per cent. in excess, though its investments are really paying better than those of the first company.

From the above it will be seen that the average policy-holder can get no valuable information from this column of percentages. The old-fashioned common-sense tables that show the average rate of interest earned by each company on its entire assets are far more illuminating as to the interest earnings of the various companies.

In the third, or mortality column, “A,” “B,” “D,” and “E” have been placed alike at 80 per cent., as being approximately the percentage of “expected” mortality that is generally experienced by a company after the effect of “selection” has mostly disappeared as to the majority of its insured. “C’s” business is so new that most of the risks have been recently examined, and if its mortality greatly exceeded the 60 per cent. indicated some special explanation would be needed. “F” really expects a greater mortality on its Industrial insurance than the “expected” for Ordinary insurances, which is the standard used here. If the exhibit called for separate statements for the Ordinary and for the Industrial department of the company’s business the percentage would be of some value in comparisons with other companies; but as now stated, it is of no use, and, as one Industrial company may have a much larger proportion of Industrial business than another, it is not even practicable to compare Industrial companies with one another fairly by means of these percentages.



It is, therefore, clear that in some respects these percentages of death loss may be quite misleading, but they are less so than is sometimes the case where companies are compared by taking the percentage of death loss to mean amount at risk. When companies are compared on this latter basis it is tacitly assumed that the probable death loss per thousand dollars is the same for all companies, which, of course, is not the case, as an old company whose policy-holders have an average age of 55 would naturally have a higher death-rate than a newer company in which the average age was only 45.

As to "Percentage of Reserves Returned on Surrenders and Lapses:"—"A" and "B" stand together at 85 per cent., this representing about the usual percentage in companies which have a considerable number of policies long in force and subject to but small surrender charges. "C's" business is on the average so new that the percentage of allowance is largely affected by the comparatively great proportion of lapses during the first two policy years when no surrender value is given. "D," a stock company, allows somewhat smaller surrender values, on the average, than would be given by companies having participating policies principally. "E," with the smaller preliminary-term reserves, is forced by competition to allow a large percentage of these reserves on surrenders, the resulting values, however, being perhaps smaller on the average than they would be on non-participating policies with full reserves. With "F" we again meet the peculiar conditions of a business partly Industrial and partly Ordinary. It is well understood that only by appropriating a considerable part of the reserves on lapsed policies can the Industrial companies reimburse themselves for the large initial outlays. The percentage shown (45 per cent.) hardly serves to give more than a vague impression of the return on the reserves in such cases. The figures that will appear in this column depend on the relative proportions of Industrial and Ordinary insurance in force in the company and the relative rapidity of recent growth in each branch of the business. To be of any value at all in connection with Industrial companies these percentages should be stated separately as to their two departments, Ordinary and Industrial.

Any peculiarity in a company's policies or any unusual method of meeting expenses would further tend to reduce the value of



the "Gain and Loss" percentages as a basis of comparison. Thus, a company whose policies are mostly on the endowment plan is almost certain to experience a lighter mortality than one in which the business is more entirely on life plans. This is an actuarial fact which could not be known to the general public. If a company holds higher reserves on some policies than originally contemplated, it will show a higher percentage in the second column—as to interest—than would otherwise be expected, and its surrender values will appear as a smaller percentage, than the normal, of these higher reserves. This special condition applies to several companies, but is little appreciated outside of the home offices. In like manner, a company doing a large proportion of sub-standard business will show, even if newly organized, a high percentage of mortality on the standard used in the exhibit; but no note is given to explain it. Differences in methods of valuation, or in arrangements for expenses, etc., would make further unexplained variations in the relative percentages.

In the preceding paragraphs it has been shown how very necessary it is to know and keep in mind a considerable number of outside facts in order to draw anything like correct conclusions from the Gain and Loss Exhibit. Therefore, as the Gain and Loss Exhibit now stands, it conceals facts quite as important as those it presents, so that, instead of furnishing an impartial and rational basis for comparing the operations of different companies, it merely serves as a means of misrepresentation. For these reasons it is the opinion of nearly all persons who are thoroughly versed in the business that no such imperfect and misleading tabulation should be published under the authority and with the apparent approval of the State insurance officials.

## CHAPTER XIX.

## ASSESSMENT AND FRATERNAL INSURANCE.

THE fundamental idea with Assessment organizations when they first appeared in this country was to do without reserve funds and to assess surviving members so as to provide for the families of those who died, and the same principle was adopted by many social and trade organizations that wished to provide insurance for their members. When the sole ostensible object is to provide insurance the organization is called an "Assessment Company," and in the cases where the members are linked together by social arrangements so that insurance is only an incident, the association is known as a "Fraternal Society."

In England and in several European countries voluntary associations for mutual aid appear to have existed from the earliest times. We have little knowledge of their primeval modes of operation, but it would seem most likely that they did not originally provide funds in advance against misfortunes, but waited until cases of need actually arose; afterwards, however, they began to lay up funds as they have done, or tried to do, for generations past.

In this country at the first the members on entrance paid a small initiation fee, but nothing in the nature of an advance premium for insurance, and no payment of that kind would be made until one of the members died, when all the survivors would be asked to contribute towards providing for his family. Originally the usual payment by each member at a death was one dollar, and the amount to be received by the stricken family depended upon the number of members and whether they honored the requisition. These crude beginnings were followed by many changes and improvements in method, so that now the original idea of *post-mortem* assessments has almost entirely disappeared.

These organizations are generally conducted very economically. This fact has enabled them to survive many vicissitudes, and now constitutes their only reason for existence, except in the cases where the members are held together by other ties than their common insurance interests.

If these organizations are well managed, they are sure to endure as long as they are conducted with the great economy that has heretofore generally characterized them.

At the International Actuarial Congress held in New York in 1903 the senior ex-president of the Actuarial Society of America, who had had personal acquaintance with the origin and development of these organizations in this country, gave a clear and comprehensive account of them, which may properly be quoted here. He said:

"The rapid increase of the assets of the regular companies led many to believe that the premiums were unnecessarily high, and about 1865 this impression began to prompt the formation of associations which proposed to give insurance for less than half the usual charges. These societies were at first much encouraged by the low death-rate that always characterizes new organizations, and which continued for several years owing to their rapidly increasing membership. The fallaciousness of their arguments was just beginning to be seen when the failure of many of the regular companies inspired distrust of the whole regular system, and gave the irregular organizations a further impetus, which lasted for many years. The unaccommodating and illiberal management of many of the regular companies prior to 1880 caused great dissatisfaction and also contributed much to the success of their rivals.

"Assessment companies are peculiar to the United States (and Canada). They formerly claimed to be similar to the English friendly societies, whose long existence, they said, proved that they also would stand the test of time. Cornelius Walford stated at the time, however, that they had no real similarity to the English societies, which made periodical collections of fixed amounts and accumulated reserves, while the original American organizations did not collect definite sums periodically nor accumulate reserves. The earliest organizations admitted persons of all ages—under, say, forty-five or fifty—on the same terms, and when a death occurred collected equally from each member without distinction as to age. The proceeds of this 'assessment,' less expenses, were paid to the family of the deceased. At first no definite amount was guaranteed, but only the proceeds of the collection, and it was generally provided that any excess of collection over a maximum sum, generally \$1,000, should be held as a reserve against the next death. After a few years some of these co-operatives, as they were also called, began to see the injustice of admitting all—old and young—on the same terms, and began to assess for varying amounts according to the age at entry, but regardless of the age attained after entry. These societies made great boasts of scientific management and published bewildering tables prepared by their pseudo 'actuaries' to prove their claims. At length, when the numbers of elderly members began to be large, it dawned upon the younger ones that it was grossly unjust to them to assess current death losses regardless of the attained ages of the survivors, and societies were then formed in which it was proposed to assess according to age attained, on what was sometimes called 'the natural



premium plan.' The promoters of these organizations made great claims to scientific management, and in many cases were honest; but many were not, and juggled with figures and technical terms in a way that imposed upon vast numbers.

"A large number of these assessment concerns were started—probably several thousand—for there were about two hundred and fifty located in one State alone. Some of them did not operate outside their own State, or even the neighboring counties, while a few spread all over the United States and even entered foreign countries.

"Many of these companies were launched by mere adventurers, who took advantage of the laxity of the laws and the mania for 'cheap insurance.' The worst of these were known as 'graveyard' companies, because they fostered speculative insurances upon persons on the brink of the grave. Pennsylvania was the principal scene of their operations, over two hundred such companies having been organized there despite the efforts of the State Insurance Commissioner; their career, however, was brief.\* Another reprehensible side growth of the assessment plan was the insurance club system, by which a number of persons would insure their lives for the benefit of their survivors in the group. Thus ten men would each take policies of \$1,000 each, and when the first died the proceeds of his policy were divided among the nine survivors; at the next death, \$1,000 was divided among the remaining eight, and so on to the end, the last survivor having the right to name a beneficiary for his policy. This form of gambling also flourished particularly in Pennsylvania and existed much longer than the 'graveyard' system.

"Many assessment associations were organized by upright though ignorant men, but in a large proportion of cases these co-operatives merited the title of 'co-duperatives,' given them by Elizur Wright, for, in many cases, both managers and members refused to acknowledge the stern logic of actual experience until too late for their salvation. The managers of many companies, however, discovered their mistakes in time, and with more or less actuarial advice have in varying degrees improved their systems, until some of them are hardly distinguishable from regular companies, and a few have lately reincorporated as such.

"Along with the business companies on the assessment plan may be included the so-called Stipulated Premium Companies, as there is no definite line of demarcation and the distinction is only in name.

"In these associations, as well as in the fraternal orders described hereafter, all naturally goes well during a few decades, provided their management is economical and their membership increases steadily in a geometrical ratio; but after a while their very size becomes a source of danger, as it is no longer practicable to obtain a sufficient number of new and young members to keep down the average death-rate, and then, unless their system is really scientific, the increase in the assessments causes dissatisfaction, loss of confidence, and in the end dissolution or reorganization.

"Fraternal organizations for insurance purposes arose about the same time as the assessment companies, and employed the same insurance system. The

\*For a very interesting description of these companies, see "Insurance and Crime," published by the Putnams, N. Y.



principal difference between them is that the assessment companies employ agents to solicit business, and in general push their business very much the same as the regular companies, while the fraternal orders generally pay—or claim to pay—nothing to obtain members, relying on personal friendship and social influences instead. Then, too, the control of assessment companies is generally in the hands of a small number of persons, while the fraternal orders aim at perfect mutuality, each member of a lodge having a vote in connection with its management and also in the selection of the delegate chosen to represent the lodge in the meeting of the council of the supreme lodge. These features have made them very popular; they have rapidly outstripped the business organizations and have become formidable competitors of the regular companies among all classes of people that do not carry large amounts of insurance.

“Many of these societies have become sensible of the defects of their original systems and have sought actuarial advice; but very few have had the intelligence and courage to make thorough reforms, and some have adopted half-way measures, which will greatly complicate the problem of rectification hereafter.”

Every year representatives from all Assessment and Fraternal organizations meet in convention. This has done much to educate their officials in the principles underlying the proper management of an insurance business. The greatest obstacle to the correction of their mistakes in the past was their inveterate prejudice against “regular” insurance companies and scientific methods. Now, however, that they have begun to adopt such methods themselves, material improvement may be expected among them.

## CHAPTER XX.

## MISCELLANEOUS TABLES AND EXPLANATIONS.

IN this chapter we take up several subjects of a general character which have a more or less direct bearing on practical life insurance.

PROBABILITIES OF LIVING AND DYING:—Often it is of interest to know what is the probability that a person of a certain age will live a year or some term of years, apart from any contract for the payment of money. For practical purposes the most convenient way of conveying this information is by stating the number that will probably be living at the close of the designated period out of one hundred persons of that age living now.

The percentages in the table on the next page are based on the American Experience Table of Mortality, and were obtained by dividing the tabular number living at the end of the term by the number at the beginning, the formula being  $\frac{l_{x+n}}{l_x}$  in which  $x$  is the present age and  $n$  the number of years in the term.

The probabilities shown in this table apply to persons in fair average health; if a man has just passed a medical examination, his probability of survivorship would be greater than shown in the table, for reasons already explained in a previous chapter, and conversely the probability would be less for a person in poor health.

To find the probability that a person will die before the end of a term of years, deduct the probability of his surviving from 100 and the balance will show the likelihood of death sometime during the term.

*Percentage of Persons Living at a Certain Age that will Survive to the End of Various Periods.*

AGE.	PERCENTAGE SURVIVING TO END OF					AGE.
	1 Year.	5 Years.	10 Years.	15 Years.	20 Years.	
21	99.21	96.08	92.18	88.23	84.15	21
22	99.21	96.06	92.11	88.11	83.96	22
23	99.20	96.03	92.05	88.00	83.77	23
24	99.20	96.00	91.98	87.87	83.55	24
25	99.19	95.97	91.90	87.73	83.31	25
26	99.19	95.93	91.82	87.57	83.05	26
27	99.18	95.89	91.73	87.41	82.76	27
28	99.17	95.86	91.63	87.23	82.45	28
29	99.17	95.81	91.53	87.03	82.09	29
30	99.16	95.76	91.41	86.81	81.70	30
31	99.15	95.71	91.29	86.57	81.26	31
32	99.14	95.66	91.15	86.31	80.76	32
33	99.13	95.60	91.00	86.01	80.21	33
34	99.12	95.53	90.83	85.68	79.59	34
35	99.11	95.46	90.65	85.31	78.91	35
36	99.09	95.38	90.45	84.90	78.14	36
37	99.08	95.29	90.22	84.43	77.29	37
38	99.06	95.19	89.97	83.90	76.34	38
39	99.04	95.08	89.69	83.32	75.30	39
40	99.02	94.96	89.37	82.66	74.15	40
41	99.00	94.83	89.01	81.93	72.89	41
42	98.97	94.68	88.60	81.11	71.50	42
43	98.95	94.52	88.14	80.20	69.98	43
44	98.92	94.33	87.63	79.20	68.32	44
45	98.88	94.11	87.04	78.08	66.52	45
46	98.84	93.86	86.39	77.86	64.57	46
47	98.80	93.58	85.66	75.51	62.47	47
48	98.75	93.26	84.85	74.04	60.22	48
49	98.69	92.90	83.96	72.42	57.81	49
50	98.62	92.49	82.97	70.68	55.25	50
51	98.55	92.04	81.88	68.80	52.55	51
52	98.46	91.54	80.69	66.76	49.72	52
53	98.37	90.99	79.39	64.57	46.77	53
54	98.26	90.38	77.97	62.23	43.74	54
55	98.14	89.71	76.42	59.74	40.64	55
56	98.01	88.96	74.74	57.09	37.50	56
57	97.87	88.15	72.93	54.31	34.35	57
58	97.71	87.25	70.97	51.40	31.20	58
59	97.53	86.27	68.86	48.39	28.07	59
60	97.33	85.19	66.59	45.30	24.99	60

EXPECTATION OF LIFE:—According to the mortality table,  $l_x$  persons are living at age  $x$ ,  $l_{x+1}$  at age  $x+1$ , and so on. Therefore, of  $l_x$  persons living now,  $l_{x+1}$  will a year hence have each lived one year, or  $l_{x+1}$  years in the aggregate:  $l_{x+2}$  will be alive two years hence and will have lived an additional  $l_{x+2}$  years. The  $l_{x+3}$  survivors of the next year will have each lived one year further, the number to be added for the third year being  $l_{x+3}$  years. If we continue to add the years of life lived by the survivors each successive year we have the total number of full years of life lived by the original  $l_x$  persons and their survivors. The total would be  $l_{x+1} + l_{x+2} + l_{x+3} + \text{etc.}$ , to table limit. If this total be divided by the original number of persons  $l_x$  we have the average number of complete years lived by each. The algebraic expression is:—

$$\frac{l_{x+1} + l_{x+2} + l_{x+3} + \text{etc. to the end of the table}}{l_x}.$$

The formula just given fails to include the fractions of the year of death lived by the members of the group. Some will just have begun a year when death comes, and others will have nearly finished a year. For practical purposes we may then consider that on the average the deaths occur in the middle of each year, and thus each one who dies lives half of the year in which death occurs. As we have followed out the group of  $l_x$  persons until all have died, therefore  $\frac{1}{2} l_x$  years are lived by the group besides the complete years included in the previous formula, and  $\frac{1}{2} \frac{l_x}{l_x}$ , or  $\frac{1}{2}$  is the addition to be made to the previous average.

The first expression, which gives the “average after-lifetime” by complete years, is called the Curtate Expectation of Life. If we add  $\frac{1}{2}$  year to this we have what is known as the Complete Expectation of Life. The formula for this is then:

$$\frac{1}{2} + \frac{l_{x+1} + l_{x+2} + l_{x+3} + \text{etc. to the end of the table}}{l_x}.$$

The complete expectation of life for each age from 21 to 80 is given in the accompanying table.



*"Expectation of Life" by American Experience Table.*

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
21	41.53	41	27.45	61	13.47
22	40.85	42	26.72	62	12.86
23	40.17	43	26.00	63	12.26
24	39.49	44	25.27	64	11.67
25	38.81	45	24.54	65	11.10
26	38.12	46	23.81	66	10.54
27	37.43	47	23.08	67	10.00
28	36.73	48	22.36	68	9.47
29	36.03	49	21.63	69	8.97
30	35.33	50	20.91	70	8.48
31	34.63	51	20.20	71	8.00
32	33.92	52	19.49	72	7.55
33	33.21	53	18.79	73	7.11
34	32.50	54	18.09	74	6.68
35	31.78	55	17.40	75	6.27
36	31.07	56	16.72	76	5.88
37	30.35	57	16.05	77	5.49
38	29.62	58	15.39	78	5.11
39	28.90	59	14.74	79	4.74
40	28.18	60	14.10	80	4.39

It must be remembered, however, that the "expectation of life" cannot properly be employed in any ordinary life insurance calculations, though many novices have done so. It is simply an average and cannot be used in computations involving compound interest. Its name is misleading.

**VIE PROBABLE:**—This French term is another expression which is very liable to be misunderstood, like the preceding one. The "Vie Probable" or "Probable Lifetime" is merely the number of years which a person has an even chance of surviving. It is found by noting at what age the number living by the mortality table is reduced to one-half of what it was at the age under consideration. Thus, of the 91,914 living at age 21, by the American Table, 47,361 will survive to age 66 and 45,291 to age 67; therefore, the "Vie Probable" will be between 45 and 46 years, or about 4 years more than the "Expectation of Life" at the same age.

**ANNUITY-CERTAIN TABLES:**—In addition to the tables previously described, showing respectively the amount, and the present value of \$1, at compound interest for various periods,

there are given two other tables derived from them. These are the amount and the present value, respectively, of \$1 *per annum* for corresponding periods of years, and they are found by summation of the above-mentioned tables. A periodical payment to be made for a term of years independently of the continuation of any life is called an Annuity-Certain.

Thus, if an annuity of \$1 is to be paid for three years "certain," the first payment being due immediately,—and it is desired to know the amount thereof at compound interest at the end of the term, it will be seen that the first dollar will accumulate for 3 years, the second dollar for 2 years, and the third dollar for one year. Therefore, if we take the sum of the first three values in the column of "amount of \$1" we have the accumulated amount of the 3-year "annuity-certain."

The present value of an annuity-certain is derived from the table of present values of \$1 in a similar manner, the first payment under the annuity in this case, however, being due a year hence.

Annuity-certain tables may be used to compare results of accumulation, etc., where no life contingency is involved, with similar matters involving such contingencies. The annuity-certain must, however, not be used in connection with the Expectation of Life to take the place of a regular life annuity, for this will not yield an equivalent result. Thus, at age 40, on American  $3\frac{1}{2}$  per cent., the present value of a life annuity of \$1 is \$16.45; the present value of an annuity-certain for 28 years, which is the "Expectation" at that age, is \$17.67, or more than a dollar greater. An annuity-certain for the "Expectation," in fact, exceeds in present value an annuity for life, at any age. The reason for this difference is that compound interest problems cannot be solved by a computation based on the average time. Thus, suppose \$100 is to be received at the end of 10 years, \$100 at the end of 20 years and \$100 at the end of 30 years, we cannot find the present value of all three amounts by assuming it to be the same as that of \$300 at the end of 20 years, though that is the average time; for in that way on the basis of four per cent. the value would apparently be \$136.92, while it really is \$144.03, or the sum of the values of each separate amount discounted for its own term.

## LIST OF TABLES.

	PAGE.
MONETARY VALUES on 3 per cent., $3\frac{1}{2}$ per cent., 4 per cent., 5 per cent. and 6 per cent. ....	161-165
ACTUARIES OR COMBINED EXPERIENCE TABLE WITH INTEREST AT 4 PER CENT.,	
Commutation Columns.....	193
Life Annuity.....	166
Net Premiums.....	195
Net Reserves, Ordinary Whole Life.....	196
Net Reserves, 20 Payment Life.....	199
Net Reserves, 20 Year Endowment.....	202
Valuation Columns.....	205
AMERICAN EXPERIENCE TABLE WITH INTEREST AT 3 PER CENT.,	
Commutation Columns.....	167
Life Annuity.....	166
Net premiums.....	169
Net Reserves, Ordinary Whole Life.....	170
Net Reserves, 20 Payment Life.....	173
Net Reserves, 20 Year Endowment.....	176
Valuation Columns.....	179
AMERICAN EXPERIENCE TABLE WITH INTEREST AT $3\frac{1}{2}$ PER CENT.,	
Commutation Columns.....	180
Life Annuity.....	166
Net Premiums.....	182
Net Reserves, Ordinary Whole Life.....	183
Net Reserves, 20 Payment Life.....	186
Net Reserves, 20 Year Endowment.....	189
Valuation Columns.....	192

*Interest Tables, Three per Cent.*

YEARS.	Amount of One Dollar at end of $n$ years.	Present Value of One Dollar due $n$ years hence $=v^n$ .	Amount of One Dollar per annum at end of $n$ years.	Present Value of One Dollar per annum for $n$ years.
1	1.0300	.970874	1.0300	.9709
2	1.0609	.942596	2.0909	1.9135
3	1.0927	.915142	3.1836	2.8286
4	1.1255	.888487	4.3091	3.7171
5	1.1593	.862609	5.4684	4.5797
6	1.1941	.837484	6.6625	5.4172
7	1.2299	.813092	7.8923	6.2303
8	1.2668	.789409	9.1591	7.0197
9	1.3048	.766417	10.4639	7.7861
10	1.3439	.744094	11.8078	8.5302
11	1.3842	.722421	13.1920	9.2526
12	1.4258	.701380	14.6178	9.9540
13	1.4685	.680951	16.0863	10.6350
14	1.5126	.661118	17.5989	11.2961
15	1.5580	.641862	19.1569	11.9379
16	1.6047	.623167	20.7616	12.5611
17	1.6528	.605016	22.4144	13.1661
18	1.7024	.587395	24.1169	13.7535
19	1.7535	.570286	25.8704	14.3238
20	1.8061	.553676	27.6765	14.8775
21	1.8603	.537549	29.5368	15.4150
22	1.9161	.521893	31.4529	15.9369
23	1.9736	.506692	33.4265	16.4436
24	2.0328	.491934	35.4593	16.9355
25	2.0938	.477606	37.5530	17.4131
26	2.1566	.463695	39.7096	17.8768
27	2.2213	.450189	41.9309	18.3270
28	2.2879	.437077	44.2189	18.7641
29	2.3566	.424346	46.5754	19.1885
30	2.4273	.411987	49.0027	19.6004
31	2.5001	.399987	51.5028	20.0004
32	2.5751	.388337	54.0778	20.3888
33	2.6523	.377026	56.7302	20.7658
34	2.7319	.366045	59.4621	21.1318
35	2.8139	.355383	62.2759	21.4872
36	2.8983	.345032	65.1742	21.8323
37	2.9852	.334983	68.1594	22.1672
38	3.0748	.325226	71.2342	22.4925
39	3.1670	.315754	74.4013	22.8082
40	3.2620	.306557	77.6633	23.1148
41	3.3599	.297628	81.0232	23.4124
42	3.4607	.288959	84.4839	23.7014
43	3.5645	.280543	88.0484	23.9819
44	3.6715	.272372	91.7199	24.2543
45	3.7816	.264439	95.5015	24.5187
46	3.8950	.256737	99.3965	24.7754
47	4.0119	.249259	103.4084	25.0247
48	4.1323	.241999	107.5406	25.2667
49	4.2562	.234950	111.7969	25.5017
50	4.3839	.228107	116.1808	25.7298



*Interest Tables, Three and One-Half per Cent.*

YEARS.	Amount of One Dollar at end of $n$ years.	Present Value of One Dollar due $n$ years hence $= v^n$ .	Amount of One Dollar per annum at end of $n$ years.	Present Value of One Dollar per annum for $n$ years.
1	1.0350	.966184	1.0350	.9662
2	1.0712	.933511	2.1062	1.8997
3	1.1087	.901943	3.2149	2.8016
4	1.1475	.871442	4.3625	3.6731
5	1.1877	.841973	5.5502	4.5151
6	1.2293	.813501	6.7794	5.3286
7	1.2723	.785991	8.0517	6.1145
8	1.3168	.759412	9.3685	6.8740
9	1.3629	.733731	10.7314	7.6077
10	1.4106	.708919	12.1420	8.3166
11	1.4600	.684946	13.6020	9.0016
12	1.5111	.661783	15.1130	9.6633
13	1.5640	.639404	16.6770	10.3027
14	1.6187	.617782	18.2957	10.9205
15	1.6753	.596891	19.9710	11.5174
16	1.7340	.576706	21.7050	12.0941
17	1.7947	.557204	23.4997	12.6513
18	1.8575	.538361	25.3572	13.1897
19	1.9225	.520156	27.2797	13.7098
20	1.9898	.502566	29.2695	14.2124
21	2.0594	.485571	31.3289	14.6980
22	2.1315	.469151	33.4604	15.1671
23	2.2061	.453286	35.6665	15.6204
24	2.2833	.437957	37.9499	16.0584
25	2.3632	.423147	40.3131	16.4815
26	2.4460	.408838	42.7591	16.8904
27	2.5316	.395012	45.2906	17.2854
28	2.6202	.381654	47.9108	17.6670
29	2.7119	.368748	50.6227	18.0358
30	2.8068	.356278	53.4295	18.3920
31	2.9050	.344230	56.3345	18.7363
32	3.0067	.332590	59.3412	19.0689
33	3.1119	.321343	62.4532	19.3902
34	3.2209	.310476	65.6740	19.7007
35	3.3336	.299977	69.0076	20.0007
36	3.4503	.289833	72.4579	20.2905
37	3.5710	.280032	76.0289	20.5705
38	3.6960	.270562	79.7249	20.8411
39	3.8254	.261413	83.5503	21.1025
40	3.9593	.252572	87.5095	21.3551
41	4.0978	.244031	91.6074	21.5991
42	4.2413	.235779	95.8486	21.8349
43	4.3897	.227806	100.2383	22.0627
44	4.5433	.220102	104.7817	22.2828
45	4.7024	.212659	109.4840	22.4955
46	4.8669	.205468	114.3510	22.7009
47	5.0373	.198520	119.3883	22.8994
48	5.2136	.191806	124.6018	23.0912
49	5.3961	.185320	129.9979	23.2766
50	5.5849	.179053	135.5828	23.4556

*Interest Tables, Four per Cent.*

YEARS.	Amount of One Dollar at end of $n$ years.	Present Value of One Dollar due $n$ years hence $=v^n$ .	Amount of One Dollar per annum at end of $n$ years.	Present Value of One Dollar per annum for $n$ years.
1	1.0400	.961538	1.0400	.9615
2	1.0816	.924556	2.1216	1.8861
3	1.1249	.888996	3.2465	2.7751
4	1.1699	.854804	4.4163	3.6299
5	1.2167	.821927	5.6330	4.4518
6	1.2653	.790315	6.8983	5.2421
7	1.3159	.759918	8.2142	6.0021
8	1.3686	.730690	9.5828	6.7327
9	1.4233	.702587	11.0061	7.4353
10	1.4802	.675564	12.4864	8.1109
11	1.5395	.649581	14.0258	8.7605
12	1.6010	.624597	15.6268	9.3851
13	1.6651	.600574	17.2919	9.9856
14	1.7317	.577475	19.0236	10.5631
15	1.8009	.555265	20.8245	11.1184
16	1.8730	.533908	22.6975	11.6523
17	1.9479	.513373	24.6454	12.1657
18	2.0258	.493628	26.6712	12.6593
19	2.1068	.474642	28.7781	13.1339
20	2.1911	.456387	30.9692	13.5903
21	2.2788	.438834	33.2480	14.0292
22	2.3699	.421955	35.6179	14.4511
23	2.4647	.405726	38.0826	14.8568
24	2.5633	.390121	40.6459	15.2470
25	2.6658	.375117	43.3117	15.6221
26	2.7725	.360689	46.0842	15.9828
27	2.8834	.346817	48.9676	16.3296
28	2.9987	.333477	51.9663	16.6631
29	3.1187	.320651	55.0849	16.9837
30	3.2434	.308319	58.3283	17.2920
31	3.3731	.296460	61.7015	17.5885
32	3.5081	.285058	65.2095	17.8736
33	3.6484	.274094	68.8579	18.1476
34	3.7943	.263552	72.6522	18.4112
35	3.9461	.253415	76.5983	18.6646
36	4.1039	.243669	80.7022	18.9083
37	4.2681	.234297	84.9703	19.1426
38	4.4388	.225285	89.4091	19.3679
39	4.6164	.216621	94.0255	19.5845
40	4.8010	.208289	98.8265	19.7928
41	4.9931	.200278	103.8196	19.9931
42	5.1928	.192575	109.0124	20.1856
43	5.4005	.185168	114.4129	20.3708
44	5.6165	.178046	120.0294	20.5488
45	5.8412	.171198	125.8706	20.7200
46	6.0748	.164614	131.9454	20.8847
47	6.3178	.158283	138.2632	21.0429
48	6.5705	.152195	144.8337	21.1951
49	6.8333	.146341	151.6671	21.3415
50	7.1067	.140713	158.7738	21.4822

*Interest Tables, Five per Cent.*

YEARS.	Amount of One Dollar at end of $n$ years.	Present Value of One Dollar due $n$ years hence $= v^n$ .	Amount of One Dollar per annum at end of $n$ years.	Present Value of One Dollar per annum for $n$ years.
1	1.0500	.952381	1.0500	.9524
2	1.1025	.907029	2.1525	1.8594
3	1.1576	.863838	3.3101	2.7232
4	1.2155	.822702	4.5256	3.5460
5	1.2763	.783526	5.8019	4.3295
6	1.3401	.746215	7.1420	5.0757
7	1.4071	.710681	8.5491	5.7864
8	1.4775	.676839	10.0266	6.4632
9	1.5513	.644609	11.5779	7.1073
10	1.6289	.613913	13.2068	7.7217
11	1.7103	.584679	14.9171	8.3064
12	1.7959	.556837	16.7130	8.8633
13	1.8856	.530321	18.5986	9.3936
14	1.9799	.505068	20.5786	9.8986
15	2.0789	.481017	22.6575	10.3797
16	2.1829	.458112	24.8404	10.8378
17	2.2920	.436297	27.1324	11.2741
18	2.4066	.415521	29.5390	11.6896
19	2.5270	.395734	32.0660	12.0853
20	2.6533	.376889	34.7193	12.4622
21	2.7860	.358942	37.5052	12.8212
22	2.9253	.341850	40.4305	13.1630
23	3.0715	.325571	43.5020	13.4886
24	3.2251	.310068	46.7271	13.7986
25	3.3864	.295303	50.1135	14.0939
26	3.5557	.281241	53.6691	14.3752
27	3.7335	.267848	57.4026	14.6430
28	3.9201	.255094	61.3227	14.8981
29	4.1161	.242946	65.4388	15.1411
30	4.3219	.231377	69.7608	15.3725
31	4.5380	.220359	74.2988	15.5928
32	4.7649	.209866	79.0638	15.8027
33	5.0032	.199873	84.0670	16.0025
34	5.2533	.190355	89.3203	16.1929
35	5.5160	.181290	94.8363	16.3742
36	5.7918	.172657	100.6281	16.5469
37	6.0814	.164436	106.7095	16.7113
38	6.3855	.156605	113.0950	16.8679
39	6.7048	.149148	119.7998	17.0170
40	7.0400	.142046	126.8398	17.1591
41	7.3920	.135282	134.2318	17.2944
42	7.7616	.128840	141.9933	17.4232
43	8.1497	.122704	150.1430	17.5459
44	8.5572	.116861	158.7002	17.6628
45	8.9850	.111297	167.6852	17.7741
46	9.4343	.105997	177.1194	17.8801
47	9.9060	.100949	187.0254	17.9810
48	10.4013	.096142	197.4267	18.0772
49	10.9213	.091564	208.3480	18.1687
50	11.4674	.087204	219.8154	18.2559

*Interest Tables, Six per Cent.*

YEARS.	Amount of One Dollar at end of $n$ years.	Present Value of One Dollar due $n$ years hence = $v^n$ .	Amount of One Dollar per annum at end of $n$ years.	Present Value of One Dollar per annum for $n$ years.
1	1.0600	.943396	1.0600	.9434
2	1.1236	.889996	2.1836	1.8334
3	1.1910	.839619	3.3746	2.6730
4	1.2625	.792094	4.6371	3.4651
5	1.3382	.747258	5.9753	4.2124
6	1.4185	.704961	7.3938	4.9173
7	1.5036	.665057	8.8975	5.5824
8	1.5938	.627412	10.4913	6.2093
9	1.6895	.591898	12.1808	6.8017
10	1.7908	.558395	13.9716	7.3601
11	1.8983	.526788	15.8699	7.8869
12	2.0122	.496969	17.8821	8.3838
13	2.1329	.468839	20.0151	8.8527
14	2.2609	.442301	22.2760	9.2950
15	2.3966	.417265	24.6725	9.7122
16	2.5404	.393646	27.2129	10.1059
17	2.6928	.371364	29.9057	10.4773
18	2.8543	.350344	32.7600	10.8276
19	3.0256	.330513	35.7856	11.1581
20	3.2071	.311805	38.9927	11.4699
21	3.3996	.294155	42.3923	11.7641
22	3.6035	.277505	45.9958	12.0416
23	3.8197	.261797	49.8156	12.3034
24	4.0489	.246979	53.8645	12.5504
25	4.2919	.232999	58.1564	12.7834
26	4.5494	.219810	62.7058	13.0032
27	4.8223	.207368	67.5281	13.2105
28	5.1117	.195630	72.6398	13.4062
29	5.4184	.184557	78.0582	13.5907
30	5.7435	.174110	83.8017	13.7648
31	6.0881	.164255	89.8898	13.9291
32	6.4534	.154957	96.3432	14.0840
33	6.8406	.146186	103.1838	14.2302
34	7.2510	.137912	110.4348	14.3681
35	7.6861	.130105	118.1209	14.4982
36	8.1473	.122741	126.2681	14.6210
37	8.6361	.115793	134.9042	14.7368
38	9.1543	.109239	144.0585	14.8460
39	9.7035	.103056	153.7620	14.9491
40	10.2857	.097222	164.0477	15.0463
41	10.9029	.091719	174.9505	15.1380
42	11.5570	.086527	186.5076	15.2245
43	12.2505	.081630	198.7580	15.3062
44	12.9855	.077009	211.7435	15.3832
45	13.7646	.072650	225.5081	15.4558
46	14.5905	.068538	240.0986	15.5244
47	15.4659	.064658	255.5645	15.5890
48	16.3939	.060998	271.9584	15.6500
49	17.3775	.057546	289.3359	15.7076
50	18.4202	.054288	307.7561	15.7619



*Value of an Annuity of One Dollar, First Payment Immediate.*

AGE.	American Experience, 3 %	American Experience, 3½ %	Acts., or Combined Expe- rience, 4 %	AGE.	American Experience, 3 %	American Experience, 3½ %	Acts., or Combined Expe- rience, 4 %
20	22.9711	21.1443	19.450	50	15.2710	14.5346	13.470
21	22.8083	21.0134	19.329	51	14.9045	14.2041	13.179
22	22.6404	20.8779	19.204	52	14.5329	13.8679	12.884
23	22.4672	20.7375	19.075	53	14.1568	13.5264	12.585
24	22.2886	20.5922	18.941	54	13.7765	13.1801	12.283
25	22.1044	20.4417	18.803	55	13.3928	12.8296	11.978
26	21.9142	20.2858	18.660	56	13.0061	12.4753	11.670
27	21.7182	20.1244	18.512	57	12.6172	12.1179	11.359
28	21.5161	19.9573	18.360	58	12.2265	11.7579	11.046
29	21.3077	19.7843	18.202	59	11.8348	11.3958	10.731
30	21.0930	19.6054	18.040	60	11.4427	11.0324	10.415
31	20.8716	19.4202	17.872	61	11.0509	10.6683	10.098
32	20.6434	19.2286	17.698	62	10.6603	10.3043	9.781
33	20.4084	19.0304	17.520	63	10.2716	9.9410	9.464
34	20.1665	18.8256	17.335	64	9.8852	9.5791	9.149
35	19.9174	18.6138	17.144	65	9.5022	9.2193	8.836
36	19.6608	18.3949	16.948	66	9.1233	8.8626	8.525
37	19.3969	18.1688	16.744	67	8.7495	8.5097	8.217
38	19.1254	17.9354	16.534	68	8.3813	8.1615	7.913
39	18.8465	17.6946	16.317	69	8.0198	7.8187	7.613
40	18.5598	17.4461	16.093	70	7.6655	7.4820	7.317
41	18.2655	17.1901	15.861	71	7.3192	7.1523	7.026
42	17.9632	16.9262	15.621	72	6.9811	6.8298	6.740
43	17.6531	16.6543	15.374	73	6.6509	6.5141	6.459
44	17.3350	16.3744	15.119	74	6.3278	6.2046	6.184
45	17.0093	16.0867	14.857	75	6.0108	5.9002	5.915
46	16.6757	15.7911	14.590	76	5.6989	5.6002	5.651
47	16.3348	15.4878	14.317	77	5.3915	5.3039	5.391
48	15.9867	15.1770	14.039	78	5.0883	5.0111	5.143
49	15.6319	14.8591	13.757	79	4.7897	4.7220	4.899
				80	4.4956	4.4368	4.661

*Commutation Columns, American Experience, Three per Cent.*

AGE.	D <sub>x</sub>	N <sub>x</sub>	C <sub>x</sub>	M <sub>x</sub>	R <sub>x</sub>
20	51290.86	1178209.61	388.6481	16974.0765	540028.16398
21	49408.31	1126918.75	376.8064	16585.4284	523054.08748
22	47592.42	1077510.44	365.3248	16208.6220	506468.65908
23	45840.91	1029918.02	354.1923	15843.2972	490260.03708
24	44151.55	984077.11	343.3984	15489.1049	474416.73988
25	42522.18	939925.56	332.9328	15145.7065	458927.63498
26	40950.75	897403.38	323.2357	14812.7737	443781.92848
27	39434.76	856452.63	313.8211	14489.5380	428969.15478
28	37972.35	817017.87	304.6807	14175.7169	414479.61678
29	36561.69	779045.52	296.2185	13871.0362	400303.89988
30	35200.56	742483.83	287.9907	13574.8177	386432.86368
31	33887.31	707283.27	279.9910	13286.8270	372858.04598
32	32620.31	673395.96	272.5900	13006.8360	359571.21898
33	31397.62	640775.65	265.7486	12734.2460	346564.38298
34	30217.37	609378.03	259.0745	12468.4974	333830.13698
35	29078.18	579160.66	252.5637	12209.4229	321361.63958
36	27978.68	550082.48	246.8824	11956.8592	309152.21668
37	26916.88	522103.80	241.3178	11709.9768	297195.35748
38	25891.58	495186.92	236.4994	11468.6590	285485.38068
39	24900.95	469295.34	231.7570	11232.1596	274016.72168
40	23943.93	444394.39	227.6854	11000.4026	262784.56208
41	23018.84	420450.46	223.6545	10772.7172	251784.15948
42	22124.74	397431.62	220.2262	10549.0627	241011.44228
43	21260.10	375306.88	217.0803	10328.8365	230462.37958
44	20423.80	354046.78	214.7242	10111.7562	220133.54308
45	19614.20	333622.98	212.5778	9897.0320	210021.78688
46	18830.34	314008.78	211.3715	9684.4542	200124.75488
47	18070.51	295178.44	210.5390	9473.0827	190440.30068
48	17333.65	277107.93	210.5155	9262.5437	180967.21798
49	16618.27	259774.28	211.4553	9052.0282	171704.67428
50	15922.79	243156.01	213.0476	8840.5729	162652.64608
51	15245.97	227233.22	215.2278	8627.5253	153812.07318
52	14586.68	211987.25	217.9353	8412.2975	145184.54788
53	13943.89	197400.57	221.1132	8194.3622	136772.25038
54	13316.65	183456.68	224.9048	7973.2490	128577.88818
55	12703.88	170140.03	229.0523	7748.3442	120604.63918
56	12104.81	157436.15	233.6946	7519.2919	112856.29498
57	11518.55	145331.34	238.5926	7285.5973	105337.00308
58	10944.47	133812.79	243.7062	7047.0047	98051.40578
59	10381.99	122868.32	249.1682	6803.2985	91004.40108

*Commutation Columns, American Experience, Three per Cent.*

AGE.	D <sub>x</sub>	N <sub>x</sub>	C <sub>x</sub>	M <sub>x</sub>	R <sub>x</sub>
60	9830.432	112486.331	254.7644	6554.1303	84201.10258
61	9289.343	102655.899	260.4633	6299.3659	77646.97228
62	8758.318	93366.556	266.0800	6038.9026	71347.60638
63	8237.140	84608.238	271.4502	5772.8226	65308.70378
64	7725.774	76371.098	276.5746	5501.3724	59535.88118
65	7224.175	68645.324	281.4546	5224.7978	54034.50878
66	6732.309	61421.149	285.6776	4943.3432	48809.71098
67	6250.543	54688.840	289.1479	4657.6656	43866.36778
68	5779.342	48438.297	291.7836	4368.5177	39208.70218
69	5319.228	42658.955	293.1361	4076.7341	34840.18448
70	4871.163	37339.727	293.1815	3783.5980	30763.45038
71	4436.103	32468.564	291.4279	3490.4165	26979.85238
72	4015.468	28032.461	287.4474	3198.9886	23489.43588
73	3611.065	24016.993	281.0950	2911.5412	20290.44728
74	3224.793	20405.928	272.4720	2630.4462	17378.90608
75	2858.395	17181.135	261.8916	2357.9742	14748.45988
76	2513.249	14322.740	249.6426	2096.0826	12390.48568
77	2190.406	11809.491	236.1901	1846.4400	10294.40308
78	1890.417	9619.085	221.7606	1610.2499	8447.96308
79	1613.596	7728.668	206.3737	1388.4893	6837.71318
80	1360.224	6115.072	190.7826	1182.1156	5449.22388
81	1129.824	4754.848	173.9759	991.3330	4267.10828
82	922.9402	3625.0237	156.1803	817.3571	3275.77528
83	739.8781	2702.0835	137.6038	661.1768	2458.41818
84	580.7245	1962.2054	119.1662	523.5730	1797.24138
85	444.6441	1381.4809	101.6860	404.4068	1273.66838
86	330.0073	936.8368	85.12296	302.72079	869.26158
87	235.2725	606.8295	69.21589	217.59783	566.54079
88	159.2040	371.5570	53.58706	148.38194	348.94296
89	100.9799	212.3530	38.80993	94.79488	200.56102
90	59.22884	111.37306	26.13805	55.98495	105.76614
91	31.36567	52.14422	16.21475	29.84690	49.78119
92	14.23735	20.77855	8.76715	13.63215	19.93429
93	5.05551	6.54120	3.60354	4.86499	6.30214
94	1.30473	1.48569	1.08577	1.26146	1.43715
95	.18096	.18096	.17569	.17569	.17569



*Premiums per \$1,000, American Experience, Three per Cent.*

AGE.	Single Premium	Whole Life.	10 Pay- ment Life.	15 Pay- ment Life.	20 Pay- ment Life.	Endow- ment 10 Years.	Endow- ment 15 Years.	Endow- ment 20 Years.
20	330.94	14.41	38.96	28.34	23.13	88.59	56.49	40.77
21	335.68	14.72	39.52	28.75	23.48	88.61	56.53	40.81
22	340.57	15.04	40.11	29.18	23.83	88.64	56.56	40.86
23	345.61	15.38	40.71	29.63	24.20	88.67	56.60	40.90
24	350.82	15.74	41.34	30.09	24.59	88.71	56.64	40.95
25	356.18	16.11	41.98	30.57	24.98	88.74	56.69	41.01
26	361.72	16.51	42.65	31.06	25.39	88.78	56.73	41.07
27	367.43	16.92	43.34	31.57	25.82	88.82	56.78	41.13
28	373.32	17.35	44.05	32.09	26.26	88.86	56.84	41.20
29	379.39	17.81	44.78	32.64	26.71	88.91	56.90	41.28
30	385.64	18.28	45.54	33.20	27.19	88.96	56.97	41.37
31	392.09	18.79	46.32	33.79	27.68	89.02	57.04	41.47
32	398.73	19.32	47.13	34.39	28.19	89.08	57.12	41.57
33	405.58	19.87	47.97	35.02	28.72	89.15	57.21	41.69
34	412.63	20.46	48.83	35.67	29.27	89.22	57.31	41.82
35	419.88	21.08	49.73	36.34	29.85	89.30	57.42	41.97
36	427.36	21.74	50.65	37.04	30.45	89.39	57.54	42.13
37	435.04	22.43	51.60	37.76	31.08	89.49	57.67	42.31
38	442.95	23.16	52.59	38.51	31.74	89.60	57.82	42.52
39	451.07	23.93	53.61	39.30	32.42	89.72	57.99	42.75
40	459.42	24.75	54.66	40.11	33.14	89.86	58.18	43.01
41	468.00	25.62	55.75	40.96	33.90	90.01	58.39	43.31
42	476.80	26.54	56.89	41.85	34.69	90.18	58.64	43.64
43	485.83	27.52	58.06	42.77	35.53	90.38	58.91	44.01
44	495.10	28.56	59.28	43.74	36.42	90.60	59.22	44.43
45	504.59	29.67	60.54	44.76	37.35	90.85	59.57	44.90
46	514.30	30.84	61.85	45.82	38.34	91.14	59.97	45.42
47	524.23	32.09	63.22	46.94	39.39	91.47	60.42	46.01
48	534.37	33.43	64.64	48.12	40.51	91.84	60.92	46.68
49	544.70	34.85	66.12	49.36	41.69	92.26	61.48	47.42
50	555.22	36.36	67.66	50.66	42.95	92.73	62.12	48.24
51	565.89	37.97	69.25	52.03	44.30	93.26	62.82	49.15
52	576.71	39.68	70.92	53.48	45.73	93.84	63.61	50.17
53	587.67	41.51	72.65	55.01	47.26	94.50	64.48	51.30
54	598.74	43.46	74.46	56.63	48.90	95.23	65.45	52.55
55	609.92	45.54	76.34	58.35	50.66	96.04	66.54	53.93
56	621.18	47.76	78.31	60.17	52.54	96.95	67.74	55.46
57	632.51	50.13	80.38	62.11	54.57	97.95	69.07	57.14
58	643.89	52.66	82.54	64.18	56.74	99.07	70.55	59.00
59	655.30	55.37	84.82	66.40	59.09	100.31	72.20	61.04
60	666.72	58.27	87.22	68.77	61.62	101.69	74.02	63.29
61	678.13	61.36	89.75	71.31	64.34	103.22	76.04	65.76
62	689.50	64.68	92.43	74.05	67.29	104.93	78.26	68.47
63	700.83	68.23	95.28	76.98	70.48	106.82	80.72	71.44
64	712.08	72.04	98.30	80.15	73.93	108.92	83.43	74.70
65	723.24	76.11	101.52	83.56	77.68	111.25	86.41	78.27
66	734.27	80.48	104.96	87.24	81.73			
67	745.16	85.17	108.62	91.21	86.12			
68	755.88	90.19	112.53	95.52	90.88			
69	766.41	95.57	116.71	100.17	96.04			
70	776.73	101.33	121.17	105.22	101.63			



*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death, American Experience, Three per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	7.09	14.40	21.94	29.71	37.73	46.01	54.54
21	7.36	14.95	22.79	30.86	39.20	47.79	56.65
22	7.65	15.54	23.68	32.07	40.73	49.66	58.86
23	7.95	16.15	24.61	33.34	42.33	51.61	61.17
24	8.27	16.80	25.59	34.66	44.01	53.64	63.57
25	8.60	17.47	26.61	36.04	45.76	55.77	66.09
26	8.94	18.17	27.68	37.48	47.58	57.99	68.71
27	9.31	18.91	28.79	38.98	49.49	60.31	71.45
28	9.69	19.67	29.95	40.56	51.48	62.73	74.31
29	10.08	20.47	31.17	42.20	53.56	65.25	77.29
30	10.49	21.31	32.45	43.92	55.73	67.90	80.41
31	10.93	22.19	33.78	45.72	58.01	70.66	83.67
32	11.39	23.11	35.17	47.60	60.39	73.54	87.05
33	11.85	24.06	36.63	49.56	62.87	76.53	90.58
34	12.35	25.08	38.16	51.62	65.46	79.67	94.27
35	12.88	26.13	39.76	53.77	68.16	82.94	98.11
36	13.42	27.23	41.42	56.00	70.97	86.34	102.12
37	14.00	28.38	43.16	58.33	73.91	89.90	106.30
38	14.58	29.57	44.96	60.77	76.98	93.61	110.65
39	15.21	30.83	46.87	63.32	80.20	97.48	115.18
40	15.86	32.14	48.85	65.99	83.54	101.52	119.88
41	16.55	33.53	50.94	68.78	87.04	105.70	124.76
42	17.26	34.97	53.11	71.68	90.65	110.03	129.79
43	18.02	36.47	55.37	74.68	94.40	114.50	134.94
44	18.79	38.03	57.70	77.78	98.25	119.07	140.21
45	19.61	39.65	60.12	80.98	102.20	123.74	145.59
46	20.44	41.32	62.60	84.24	106.21	128.50	151.05
47	21.31	43.03	65.13	87.56	110.31	133.34	156.62
48	22.20	44.77	67.70	90.94	114.47	138.25	162.26
49	23.09	46.53	70.30	94.36	118.69	143.24	167.98
50	24.00	48.33	72.96	97.86	122.99	148.31	173.78
51	24.93	50.17	75.68	101.43	127.37	153.47	179.67
52	25.88	52.05	78.45	105.06	131.82	158.70	185.66
53	26.86	53.97	81.28	108.75	136.35	164.02	191.72
54	27.85	55.92	84.15	112.51	140.95	169.41	197.84
55	28.87	57.91	87.08	116.33	145.61	174.86	204.02
56	29.90	59.94	90.06	120.21	150.33	180.36	210.25
57	30.96	62.01	93.09	124.13	155.09	185.91	216.52
58	32.04	64.11	96.15	128.10	159.90	191.49	222.82
59	33.13	66.23	99.24	132.09	164.73	197.10	229.11
60	34.23	68.37	102.35	136.11	169.58	202.69	235.37
61	35.35	70.53	105.48	140.15	174.43	208.26	241.58
62	36.47	72.71	108.64	144.18	179.25	213.79	247.70
63	37.61	74.90	111.79	148.19	184.03	219.23	253.72
64	38.75	77.07	114.90	152.14	188.71	224.55	259.59
65	39.87	79.22	117.96	156.01	193.29	229.74	265.31
66	40.98	81.34	120.96	159.80	197.75	234.81	271.00
67	42.08	83.40	123.89	163.47	202.11	239.84	276.78
68	43.13	85.41	126.72	167.06	206.45	245.01	282.83
69	44.18	87.36	129.51	170.63	210.97	250.51	289.39
70	45.18	89.28	132.35	174.50	215.86	256.55	296.66

*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death, American Experience, Three per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	63.34	72.41	81.76	91.40	101.33	111.56	122.09
21	65.79	75.21	84.91	94.91	105.22	115.83	126.75
22	68.35	78.12	88.20	98.58	109.27	120.27	131.61
23	71.02	81.17	91.64	102.40	113.49	124.91	136.66
24	73.81	84.36	95.21	106.39	117.90	129.74	141.92
25	76.72	87.67	98.94	110.55	122.49	134.77	147.39
26	79.75	91.12	102.83	114.87	127.26	139.99	153.07
27	82.92	94.73	106.88	119.38	132.23	145.43	158.98
28	86.23	98.50	111.11	124.08	137.40	151.08	165.13
29	89.68	102.42	115.51	128.96	142.78	156.96	171.52
30	93.28	106.50	120.10	134.05	148.38	163.08	178.16
31	97.03	110.76	124.87	139.35	154.21	169.45	185.05
32	100.94	115.19	129.83	144.86	160.27	176.05	192.20
33	105.00	119.81	135.01	150.60	166.56	182.90	199.60
34	109.25	124.63	140.40	156.56	173.10	190.00	207.26
35	113.68	129.65	146.01	162.76	179.87	197.35	215.16
36	118.29	134.86	151.83	169.17	186.87	204.92	223.28
37	123.09	140.29	157.86	175.81	194.10	212.71	231.60
38	128.09	145.91	164.11	182.67	201.54	220.70	240.12
39	133.27	151.74	170.57	189.72	209.16	228.88	248.84
40	138.64	157.76	177.20	196.95	216.97	237.23	257.72
41	144.19	163.95	184.01	204.35	224.94	245.76	266.77
42	149.88	170.28	190.96	211.90	233.07	254.44	275.96
43	155.70	176.75	198.06	219.60	241.34	263.24	285.27
44	161.64	183.34	205.28	227.42	249.72	272.16	294.69
45	167.70	190.06	212.62	235.35	258.22	281.18	304.22
46	173.86	196.87	220.06	243.38	266.80	290.30	313.81
47	180.11	203.78	227.59	251.50	275.49	299.49	323.47
48	186.44	210.77	235.21	259.71	284.24	308.74	333.18
49	192.86	217.85	242.91	267.99	293.05	318.04	342.91
50	199.36	225.01	250.69	276.34	301.92	327.38	352.68
51	205.96	232.27	258.55	284.76	310.84	336.76	362.46
52	212.64	239.59	266.47	293.22	319.80	346.16	372.23
53	219.39	246.98	274.44	301.73	328.79	355.55	381.96
54	226.19	254.42	282.46	310.26	337.76	364.90	391.62
55	233.05	261.90	290.50	318.79	346.70	374.19	401.19
56	239.95	269.41	298.53	327.28	355.59	383.38	410.62
57	246.89	276.91	306.54	335.72	364.38	392.46	419.90
58	253.81	284.39	314.50	344.07	373.05	401.37	429.02
59	260.70	291.81	322.36	352.29	381.55	410.12	438.02
60	267.54	299.13	330.10	360.36	389.90	418.76	447.00
61	274.29	306.35	337.69	368.28	398.16	427.39	456.09
62	280.94	313.42	345.13	376.10	406.41	436.16	465.41
63	287.43	320.35	352.49	383.95	414.81	445.18	475.11
64	293.78	327.18	359.87	391.95	423.49	454.59	485.26
65	300.06	334.07	367.43	400.25	432.61	464.51	495.93
66	306.41	341.17	375.35	409.05	442.27	475.00	507.24
67	313.01	348.66	383.79	418.44	452.57	486.18	519.00
68	320.05	356.73	392.89	428.52	463.61	497.87	531.37
69	327.73	365.53	402.76	439.43	475.24	510.25	544.62
70	336.20	375.16	413.52	450.98	487.61	523.57	559.21

*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death, American Experience, Three per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	132.94	144.11	155.60	167.42	179.56	192.04
21	138.00	149.57	161.47	173.70	186.27	199.17
22	143.26	155.25	167.57	180.23	193.23	206.59
23	148.74	161.16	173.92	187.02	200.47	214.27
24	154.44	167.30	180.50	194.06	207.98	222.25
25	160.36	173.67	187.34	201.37	215.77	230.50
26	166.50	180.29	194.44	208.96	223.83	239.05
27	172.90	187.18	201.82	216.82	232.18	247.88
28	179.54	194.32	209.47	224.97	240.81	256.99
29	186.44	201.73	217.39	233.38	249.72	266.38
30	193.61	209.42	225.58	242.08	258.90	276.02
31	201.04	217.37	234.05	251.05	268.34	285.90
32	208.72	225.58	242.77	260.25	278.00	296.00
33	216.66	234.05	251.73	269.69	287.90	306.33
34	224.86	242.76	260.93	279.35	298.00	316.86
35	233.28	251.68	270.34	289.22	308.32	327.58
36	241.92	260.82	279.95	299.29	318.81	338.48
37	250.76	270.15	289.76	309.54	329.48	349.53
38	259.79	279.68	299.74	319.96	340.29	360.72
39	269.02	289.38	309.89	330.53	351.26	372.04
40	278.40	299.23	320.19	341.24	362.34	383.47
41	287.94	309.24	330.62	352.07	373.54	394.98
42	297.61	319.36	341.17	362.99	384.80	406.55
43	307.40	329.59	351.80	374.00	396.12	418.14
44	317.29	339.91	362.51	385.04	407.47	429.75
45	327.27	350.30	373.26	396.12	418.83	441.35
46	337.30	360.73	384.04	407.21	430.18	452.90
47	347.39	371.19	394.84	418.29	441.48	464.37
48	357.49	381.66	405.62	429.32	452.70	475.73
49	367.62	392.13	416.36	440.28	463.83	486.96
50	377.76	402.57	427.05	451.16	474.84	498.04
51	387.88	412.97	437.67	461.92	485.69	508.93
52	397.96	423.29	448.17	472.55	496.37	519.63
53	407.97	433.50	458.53	482.99	506.87	530.19
54	417.87	443.58	468.72	493.26	517.22	540.68
55	427.64	453.50	478.74	503.39	527.52	551.19
56	437.25	463.24	488.63	513.47	537.85	561.83
57	446.70	472.86	498.48	523.60	548.32	572.69
58	456.02	482.45	508.38	533.89	559.04	583.83
59	465.32	492.11	518.46	544.44	570.05	595.28
60	474.71	501.96	528.83	555.32	581.42	607.12
61	484.30	512.13	539.56	566.58	593.19	619.17
62	494.25	522.68	550.70	578.28	605.22	631.56
63	504.62	533.69	562.32	590.28	617.61	644.45
64	515.47	545.22	574.27	602.67	630.55	658.19
65	526.88	557.10	586.65	615.66	644.41	673.03
66	538.71	569.49	599.70	629.64	659.45	688.84
67	551.09	582.59	613.82	644.90	675.54	705.21
68	564.26	596.85	629.30	661.29	692.26	721.54
69	578.68	612.59	646.02	678.39	708.99	737.78
70	594.68	629.66	663.52	695.54	725.66	754.69



*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty  
Equal Annual Premiums, American Experience, Three per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	16.15	32.86	50.16	68.07	86.62	105.84	125.73
21	16.46	33.49	51.12	69.38	88.29	107.87	128.14
22	16.78	34.14	52.12	70.74	90.01	109.96	130.63
23	17.11	34.82	53.15	72.13	91.78	112.13	133.19
24	17.45	35.52	54.22	73.57	93.61	114.35	135.82
25	17.81	36.24	55.31	75.06	95.49	116.64	138.54
26	18.17	36.97	56.44	76.58	97.42	119.00	141.33
27	18.55	37.74	57.60	78.15	99.42	121.43	144.20
28	18.94	38.52	58.79	79.77	101.47	123.92	147.15
29	19.33	39.33	60.02	81.43	103.57	126.48	150.19
30	19.74	40.17	61.30	83.14	105.74	129.13	153.31
31	20.17	41.03	62.60	84.90	107.98	131.83	156.51
32	20.61	41.91	63.93	86.71	110.26	134.62	159.79
33	21.05	42.81	65.32	88.57	112.62	137.47	163.15
34	21.51	43.75	66.74	90.49	115.03	140.40	166.60
35	22.00	44.72	68.20	92.46	117.52	143.40	170.14
36	22.48	45.71	69.69	94.47	120.05	146.48	173.76
37	22.99	46.72	71.23	96.53	122.66	149.64	177.49
38	23.50	47.77	72.81	98.66	125.34	152.88	181.29
39	24.04	48.84	74.44	100.85	128.10	156.20	185.19
40	24.58	49.95	76.11	103.10	130.92	159.60	189.16
41	25.16	51.10	77.85	105.42	133.82	163.08	193.20
42	25.75	52.29	79.62	107.78	136.77	166.61	197.30
43	26.36	53.50	81.44	110.20	139.78	170.19	201.42
44	26.97	54.74	83.29	112.65	142.81	173.77	205.55
45	27.62	56.00	85.17	115.13	145.86	177.37	209.67
46	28.26	57.28	87.07	117.61	148.90	180.95	213.77
47	28.92	58.58	88.97	120.09	151.93	184.51	217.85
48	29.58	59.87	90.86	122.55	154.94	188.05	221.87
49	30.23	61.15	92.73	124.99	157.93	191.55	225.86
50	30.89	62.42	94.61	127.43	160.90	195.02	229.80
51	31.54	63.71	96.49	129.87	163.87	198.47	233.70
52	32.21	65.01	98.37	132.31	166.82	201.90	237.57
53	32.88	66.31	100.27	134.75	169.76	205.30	241.39
54	33.56	67.61	102.16	137.18	172.69	208.67	245.15
55	34.24	68.93	104.06	139.62	175.61	212.02	248.86



*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, American Experience, Three per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	146.33	167.67	189.76	212.64	236.35	260.90	286.33
21	149.13	170.87	193.38	216.69	240.84	265.85	291.75
22	152.02	174.17	197.11	220.86	245.46	270.93	297.32
23	154.99	177.57	200.95	225.15	250.21	276.17	303.05
24	158.05	181.07	204.89	229.56	255.10	281.55	308.93
25	161.21	184.66	208.95	234.09	260.12	287.07	314.97
26	164.44	188.36	213.12	238.75	265.28	292.73	321.16
27	167.77	192.16	217.40	243.53	270.56	298.55	327.51
28	171.19	196.06	221.80	248.43	275.99	304.50	334.01
29	174.70	200.07	226.31	253.46	281.54	310.60	340.67
30	178.32	204.18	230.94	258.61	287.23	316.85	347.49
31	182.02	208.40	235.67	263.88	293.06	323.24	354.46
32	185.81	212.71	240.52	269.28	299.02	329.77	361.57
33	189.69	217.13	245.49	274.81	305.11	336.44	368.82
34	193.68	221.66	250.58	280.46	311.34	343.24	376.20
35	197.77	226.31	255.78	286.24	317.68	350.16	383.70
36	201.95	231.05	261.10	292.11	324.13	357.18	391.28
37	206.23	235.90	266.51	298.09	330.67	364.27	398.93
38	210.60	240.83	272.01	304.15	337.28	371.42	406.63
39	215.06	245.86	277.60	310.28	343.94	378.62	414.36
40	219.60	250.96	283.23	316.44	350.63	385.84	422.11
41	224.21	256.10	288.90	322.63	357.34	393.06	429.85
42	228.85	261.27	294.58	328.82	364.03	400.26	437.55
43	233.50	266.43	300.26	335.00	370.70	407.41	445.20
44	238.15	271.59	305.91	341.13	377.31	414.49	452.76
45	242.78	276.72	311.52	347.21	383.84	421.49	460.22
46	247.38	281.80	317.06	353.20	390.28	428.37	467.55
47	251.94	286.83	322.53	359.11	396.62	435.12	474.73
48	256.44	291.78	327.91	364.90	402.81	441.71	481.72
49	260.88	296.64	333.19	370.57	408.85	448.12	488.50
50	265.26	301.44	338.37	376.11	414.73	454.34	495.08
51	269.59	306.15	343.43	381.50	420.43	460.36	501.41
52	273.85	310.77	348.37	386.74	425.96	466.15	507.47
53	278.04	315.28	353.18	391.81	431.27	471.69	513.25
54	282.14	319.69	357.85	396.71	436.36	476.95	518.70
55	286.17	323.99	362.37	401.39	441.19	481.92	523.79

*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty  
Equal Annual Premiums, American Experience, Three per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	312.68	339.98	368.26	397.57	427.94	459.42
21	318.58	346.38	375.18	405.02	435.95	468.00
22	324.65	352.97	382.29	412.68	444.16	476.80
23	330.89	359.72	389.59	420.53	452.60	485.83
24	337.29	366.66	397.08	428.59	461.25	495.10
25	343.86	373.77	404.76	436.85	470.12	504.59
26	350.59	381.06	412.62	445.32	479.19	514.30
27	357.49	388.54	420.69	453.99	488.49	524.23
28	364.56	396.19	428.93	462.85	497.97	534.37
29	371.80	404.01	437.36	471.89	507.65	544.70
30	379.19	412.01	445.97	481.12	517.52	555.22
31	386.75	420.16	454.73	490.51	527.54	565.89
32	394.45	428.46	463.64	500.03	537.70	576.71
33	402.30	436.90	472.68	509.69	547.99	587.67
34	410.27	445.46	481.83	519.45	558.39	598.74
35	418.33	454.11	491.07	529.31	568.89	609.92
36	426.48	462.83	500.39	539.24	579.47	621.18
37	434.69	471.61	509.76	549.23	590.10	632.51
38	442.94	480.43	519.17	559.25	600.77	643.89
39	451.23	489.27	528.59	569.28	611.47	655.30
40	459.51	498.11	538.00	579.31	622.16	666.72
41	467.78	506.92	547.39	589.30	632.82	678.13
42	475.99	515.68	556.71	599.24	643.44	689.50
43	484.15	524.36	565.96	609.10	653.97	700.83
44	492.20	532.93	575.09	618.84	664.41	712.08
45	500.15	541.38	584.08	628.45	674.73	723.24
46	507.94	549.66	592.90	637.89	684.90	734.27
47	515.55	557.75	601.53	647.14	694.88	745.16
48	522.96	565.63	609.94	656.16	704.66	755.88
49	530.15	573.27	618.08	664.92	714.19	766.41
50	537.10	580.63	625.94	673.40	723.46	776.73
51	543.77	587.69	633.48	681.55	732.44	786.82
52	550.14	594.42	640.67	689.36	741.09	796.67
53	556.18	600.78	647.47	696.78	749.41	806.28
54	561.84	606.74	653.85	703.79	757.38	815.69
55	567.10	612.25	659.78	710.39	765.04	824.93

*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	34.46	70.18	107.22	145.63	185.47	226.79	269.65
21	34.45	70.17	107.20	145.61	185.44	226.76	269.61
22	34.45	70.16	107.19	145.60	185.42	226.73	269.57
23	34.44	70.16	107.19	145.59	185.41	226.71	269.54
24	34.45	70.16	107.19	145.58	185.40	226.69	269.51
25	34.45	70.16	107.19	145.58	185.39	226.67	269.49
26	34.45	70.16	107.19	145.57	185.38	226.65	269.47
27	34.45	70.17	107.19	145.57	185.38	226.65	269.45
28	34.46	70.18	107.20	145.59	185.38	226.64	269.43
29	34.46	70.19	107.21	145.60	185.39	226.64	269.43
30	34.47	70.21	107.24	145.62	185.41	226.66	269.43
31	34.49	70.23	107.27	145.65	185.44	226.68	269.45
32	34.51	70.26	107.30	145.69	185.48	226.72	269.47
33	34.52	70.29	107.35	145.75	185.54	226.77	269.51
34	34.55	70.34	107.41	145.83	185.61	226.84	269.57
35	34.59	70.40	107.50	145.91	185.71	226.93	269.66
36	34.62	70.47	107.58	146.02	185.82	227.05	269.77
37	34.67	70.55	107.70	146.15	185.97	227.21	269.93
38	34.72	70.65	107.82	146.31	186.15	227.41	270.13
39	34.78	70.76	107.99	146.52	186.39	227.66	270.38
40	34.85	70.90	108.19	146.77	186.67	227.95	270.67
41	34.95	71.08	108.44	147.06	187.01	228.30	271.01
42	35.05	71.28	108.72	147.41	187.38	228.69	271.38
43	35.18	71.51	109.04	147.79	187.80	229.12	271.78
44	35.31	71.77	109.38	148.20	188.25	229.56	272.18
45	35.48	72.05	109.78	148.66	188.73	230.02	272.59
46	35.64	72.36	110.19	149.13	189.21	230.49	273.00
47	35.82	72.69	110.61	149.61	189.71	230.96	273.41
48	36.02	73.02	111.04	150.09	190.21	231.44	273.82
49	36.21	73.36	111.48	150.59	190.73	231.92	274.24
50	36.41	73.71	111.94	151.12	191.27	232.44	274.68
51	36.62	74.10	112.45	151.69	191.87	233.00	275.15
52	36.85	74.52	113.00	152.32	192.50	233.60	275.67
53	37.11	74.96	113.59	152.99	193.20	234.27	276.24
54	37.38	75.45	114.22	153.71	193.96	234.99	276.86
55	37.68	75.98	114.92	154.52	194.80	235.79	277.54

*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.
20	314.11	360.24	408.10	457.76	509.31	562.82
21	314.06	360.18	408.03	457.69	509.24	562.73
22	314.02	360.12	407.97	457.62	509.15	562.64
23	313.98	360.08	407.91	457.55	509.07	562.56
24	313.94	360.03	407.85	457.48	508.99	562.46
25	313.91	359.98	407.79	457.41	508.90	562.37
26	313.87	359.93	407.73	457.33	508.81	562.26
27	313.84	359.89	407.67	457.26	508.72	562.15
28	313.82	359.85	407.62	457.18	508.63	562.04
29	313.80	359.82	407.56	457.11	508.53	561.92
30	313.79	359.79	407.51	457.03	508.43	561.80
31	313.78	359.77	407.47	456.97	508.34	561.69
32	313.79	359.75	407.44	456.91	508.26	561.58
33	313.81	359.76	407.41	456.86	508.18	561.47
34	313.86	359.79	407.42	456.84	508.13	561.38
35	313.94	359.85	407.45	456.84	508.08	561.28
36	314.04	359.93	407.51	456.85	508.04	561.19
37	314.19	360.05	407.59	456.88	508.02	561.09
38	314.37	360.20	407.70	456.93	507.99	560.98
39	314.60	360.40	407.84	456.99	507.96	560.85
40	314.87	360.62	407.98	457.05	507.91	560.69
41	315.18	360.86	408.14	457.09	507.84	560.50
42	315.50	361.11	408.28	457.12	507.74	560.27
43	315.83	361.35	408.41	457.13	507.61	559.99
44	316.10	361.58	408.53	457.10	507.42	559.65
45	316.50	361.81	408.62	457.04	507.19	559.24
46	316.82	362.00	408.68	456.92	506.90	558.77
47	317.13	362.20	408.70	456.77	506.55	558.22
48	317.44	362.36	408.69	456.56	506.13	557.58
49	317.74	362.51	408.66	456.31	505.64	556.85
50	318.05	362.66	408.61	456.03	505.10	556.05
51	318.40	362.83	408.55	455.71	504.51	555.17
52	318.77	363.01	408.49	455.37	503.87	554.22
53	319.19	363.21	408.43	455.02	503.18	553.18
54	319.64	363.43	408.37	454.64	502.43	552.05
55	320.14	363.69	408.33	454.23	501.62	550.81



*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three per Cent.*

AGE.	14th Year.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.
20	618.37	676.06	735.97	798.21	862.88	930.10
21	618.28	675.97	735.88	798.13	862.82	930.06
22	618.19	675.87	735.78	798.04	862.74	930.02
23	618.09	675.77	735.68	797.95	862.67	929.97
24	617.99	675.66	735.58	797.84	862.58	929.92
25	617.88	675.54	735.46	797.73	862.49	929.87
26	617.76	675.42	735.33	797.61	862.40	929.81
27	617.64	675.29	735.20	797.49	862.29	929.74
28	617.51	675.15	735.06	797.35	862.18	929.67
29	617.38	675.00	734.91	797.21	862.05	929.59
30	617.24	674.85	734.75	797.05	861.92	929.51
31	617.11	674.70	734.58	796.89	861.77	929.41
32	616.97	674.53	734.40	796.71	861.61	929.30
33	616.83	674.36	734.21	796.50	861.43	929.19
34	616.69	674.19	734.01	796.29	861.23	929.05
35	616.55	674.00	733.77	796.05	861.01	928.91
36	616.39	673.78	733.52	795.77	860.77	928.74
37	616.22	673.54	733.23	795.47	860.49	928.56
38	616.02	673.27	732.90	795.12	860.18	928.35
39	615.79	672.96	732.53	794.73	859.82	928.12
40	615.53	672.61	732.12	794.29	859.43	927.86
41	615.23	672.21	731.64	793.80	858.98	927.57
42	614.87	671.74	731.10	793.23	858.48	927.24
43	614.45	671.20	730.48	792.60	857.91	926.86
44	613.96	670.58	729.78	791.88	857.27	926.45
45	613.40	669.88	728.99	791.06	856.55	925.98
46	612.74	669.08	728.09	790.15	855.74	925.45
47	612.00	668.17	727.07	789.12	854.84	924.86
48	611.15	667.15	725.93	787.97	853.82	924.20
49	610.20	666.01	724.67	786.69	852.69	923.46
50	609.15	664.75	723.27	785.27	851.43	922.64
51	608.00	663.36	721.72	783.70	850.04	921.72
52	606.74	661.83	720.02	781.96	848.49	920.70
53	605.36	660.17	718.15	780.04	846.78	919.58
54	603.86	658.33	716.08	777.92	844.88	918.33
55	602.19	656.29	713.78	775.56	842.78	916.95

## Valuation Columns.

Am. Exp. 3 %.

$$u_x = \frac{D_x}{D_{x+1}}$$

$$k_x = \frac{C_x}{D_{x+1}}$$

AGE.	$u_x$	$k_x$	AGE.	$u_x$	$k_x$
20	1.038 102	0.007 866	60	1.058 248	0.027 425
21	1.038 155	0.007 917	61	1.060 631	0.029 739
22	1.038 209	0.007 969	62	1.063 272	0.032 303
23	1.038 263	0.008 022	63	1.066 190	0.035 136
24	1.038 318	0.008 076	64	1.069 433	0.038 285
25	1.038 374	0.008 130	65	1.073 061	0.041 807
26	1.038 443	0.008 197	66	1.077 076	0.045 704
27	1.038 512	0.008 265	67	1.081 532	0.050 031
28	1.038 583	0.008 333	68	1.086 500	0.054 855
29	1.038 668	0.008 415	69	1.091 983	0.060 178
30	1.038 753	0.008 499	70	1.098 073	0.066 090
31	1.038 841	0.008 583	71	1.104 754	0.072 576
32	1.038 942	0.008 682	72	1.111 990	0.079 602
33	1.039 058	0.008 795	73	1.119 782	0.087 167
34	1.039 177	0.008 910	74	1.128 183	0.095 323
35	1.039 298	0.009 027	75	1.137 331	0.104 204
36	1.039 447	0.009 172	76	1.147 390	0.113 971
37	1.039 600	0.009 320	77	1.158 689	0.124 941
38	1.039 783	0.009 498	78	1.171 556	0.137 433
39	1.039 970	0.009 679	79	1.186 272	0.151 720
40	1.040 188	0.009 891	80	1.203 926	0.168 861
41	1.040 412	0.010 109	81	1.224 157	0.188 502
42	1.040 669	0.010 359	82	1.247 422	0.211 089
43	1.040 948	0.010 629	83	1.274 060	0.236 952
44	1.041 276	0.010 947	84	1.306 044	0.268 004
45	1.041 628	0.011 289	85	1.347 377	0.308 133
46	1.042 048	0.011 697	86	1.402 660	0.361 806
47	1.042 511	0.012 146	87	1.477 805	0.434 762
48	1.043 048	0.012 668	88	1.576 591	0.530 671
49	1.043 678	0.013 280	89	1.704 912	0.655 254
50	1.044 393	0.013 974	90	1.888 333	0.833 333
51	1.045 198	0.014 755	91	2.203 056	1.138 889
52	1.046 098	0.015 629	92	2.816 203	1.734 177
53	1.047 102	0.016 604	93	3.874 762	2.761 905
54	1.048 235	0.017 704	94	7.210 000	6.000 000
55	1.049 490	0.018 922			
56	1.050 897	0.020 289			
57	1.052 454	0.021 800			
58	1.054 178	0.023 474			
59	1.056 107	0.025 347			

*Commutation Columns, American Experience, Three and One-Half per Cent.*

AGE.	D <sub>x</sub>	N <sub>x</sub>	C <sub>x</sub>	M <sub>x</sub>	R <sub>x</sub>
20	46556.2	984399.5	351.0	13267.5	397287.23
21	44630.8	937843.3	338.8	12916.5	384019.73
22	42782.8	893212.5	326.8	12577.7	371103.23
23	41009.2	850429.7	315.3	12250.9	358525.53
24	39307.1	809420.5	304.3	11935.6	346274.63
25	37673.6	770113.4	293.6	11631.3	334339.03
26	36106.1	732439.8	283.6	11337.7	322707.73
27	34601.5	696333.7	274.0	11054.1	311370.03
28	33157.4	661732.2	264.8	10780.1	300315.93
29	31771.3	628574.8	256.1	10515.3	289535.83
30	30440.8	596803.5	247.9	10259.2	279020.53
31	29163.5	566362.7	239.8	10011.3	268761.33
32	27937.5	537199.2	232.25	9771.45	258750.03
33	26760.5	509261.7	225.47	9539.20	248978.58
34	25630.1	482501.2	218.68	9313.73	239439.38
35	24544.7	456871.1	212.20	9095.05	230125.65
36	23502.5	432326.4	206.33	8882.85	221030.60
37	22501.4	408823.9	200.79	8676.52	212147.75
38	21539.7	386322.5	195.80	8475.73	203471.23
39	20615.5	364782.8	191.07	8279.93	194995.50
40	19727.4	344167.3	186.69	8088.96	186715.57
41	18873.6	324439.9	182.47	7902.27	178626.61
42	18052.9	305566.3	178.82	7719.80	170724.34
43	17263.6	287513.4	175.41	7540.98	163004.54
44	16504.4	270249.8	172.68	7365.57	155463.56
45	15773.6	253745.4	170.20	7192.89	148097.99
46	15070.0	237971.8	168.29	7022.69	140905.10
47	14392.1	222901.8	166.91	6854.40	133882.41
48	13738.5	208509.7	166.02	6687.49	127028.01
49	13107.9	194771.2	166.04	6521.47	120340.52
50	12498.6	181663.3	166.34	6355.43	113819.05
51	11909.6	169164.7	167.36	6189.09	107463.62
52	11339.5	157255.1	168.64	6021.73	101274.53
53	10787.4	145915.6	170.22	5853.09	95252.80
54	10252.4	135128.2	172.30	5682.87	89399.71
55	9733.40	124875.8	174.65	5510.57	83716.84
56	9229.60	115142.4	177.32	5335.92	78206.27
57	8740.17	105912.81	180.17	5158.60	72870.35
58	8264.44	97172.64	183.14	4978.43	67711.75
59	7801.83	88908.20	186.35	4795.29	62733.32

*Commutation Columns, American Experience, Three and One-Half per Cent.*

AGE.	D <sub>x</sub>	N <sub>x</sub>	C <sub>x</sub>	M <sub>x</sub>	R <sub>x</sub>
60	7351.65	81106.37	189.61	4608.94	57938.03
61	6913.44	73754.72	192.90	4419.33	53329.09
62	6486.75	66841.28	196.12	4226.43	48909.76
63	6071.27	60354.53	199.11	4030.31	44683.33
64	5666.85	54283.26	201.89	3831.20	40653.02
65	5273.33	48616.41	204.46	3629.31	36821.82
66	4890.55	43343.08	206.52	3424.85	33192.51
67	4518.65	38452.53	208.03	3218.33	29767.66
68	4157.82	33933.88	208.89	3010.30	26549.33
69	3808.32	29776.06	208.87	2801.41	23539.03
70	3470.67	25967.74	207.875	2592.539	20737.627
71	3145.43	22497.07	205.643	2384.664	18145.088
72	2833.42	19351.64	201.855	2179.021	15760.424
73	2535.75	16518.22	196.430	1977.166	13581.403
74	2253.57	13932.47	189.493	1780.736	11604.237
75	1987.87	11728.90	181.258	1591.243	9823.501
76	1739.39	9741.03	171.940	1409.985	8232.258
77	1508.63	8001.64	161.884	1238.045	6822.273
78	1295.73	6493.01	156.986	1076.161	5584.228
79	1100.65	5197.275	140.092	924.898	4508.067
80	923.338	4096.625	128.881	784.806	3583.169
81	763.234	3173.287	116.959	655.925	2798.363
82	620.465	2410.053	104.4882	538.9661	2142.4386
83	494.995	1789.588	91.6152	434.4779	1603.4725
84	386.641	1294.593	78.9562	342.8627	1168.9946
85	294.610	907.952	67.0494	263.9065	826.1319
86	217.598	613.342	55.8567	196.8571	562.2254
87	154.383	395.744	45.1993	141.0004	365.3683
88	103.963	241.3608	34.82425	95.80108	224.36790
89	65.6231	137.3978	25.09928	60.97683	128.56682
90	38.3047	71.7747	16.82248	35.87755	67.58999
91	20.1869	33.4700	10.38536	19.05507	31.71244
92	9.11889	13.28309	5.588164	8.669706	12.657379
93	3.22236	4.164203	2.285780	3.081542	3.987673
94	.827611	.941843	.685393	.795762	.906131
95	.114232	.114232	.110369	.110369	.110369



*Net Premiums per \$1,000, American Experience, Three and One-Half per Cent.*

AGE.	Single Pre- mium.	Whole Life.	10 Pay- ment Life.	15 Pay- ment Life.	20 Pay- ment Life.	Endow- ment 10 Years.	Endow- ment 15 Years.	Endow- ment 20 Years.
20	284.97	13.48	34.23	25.15	20.72	86.30	54.44	38.90
21	289.40	13.77	34.77	25.55	21.06	86.33	54.47	38.94
22	293.99	14.08	35.33	25.97	21.40	86.36	54.51	38.99
23	298.73	14.41	35.91	26.40	21.76	86.39	54.55	39.04
24	303.65	14.75	36.51	26.84	22.14	86.42	54.59	39.09
25	308.73	15.10	37.13	27.31	22.53	86.45	54.63	39.14
26	314.01	15.48	37.78	27.79	22.93	86.49	54.68	39.20
27	319.47	15.88	38.45	28.29	23.35	86.53	54.73	39.27
28	325.12	16.29	39.14	28.81	23.79	86.58	54.79	39.34
29	330.97	16.73	39.86	29.35	24.24	86.63	54.85	39.42
30	337.02	17.19	40.61	29.91	24.71	86.68	54.92	39.51
31	343.28	17.68	41.38	30.49	25.21	86.73	54.99	39.61
32	349.76	18.19	42.19	31.09	25.72	86.80	55.07	39.72
33	356.46	18.73	43.02	31.72	26.25	86.86	55.16	39.83
34	363.39	19.30	43.88	32.37	26.81	86.94	55.26	39.97
35	370.55	19.91	44.78	33.05	27.40	87.02	55.37	40.12
36	377.95	20.55	45.70	33.75	28.01	87.11	55.49	40.28
37	385.60	21.22	46.67	34.49	28.64	87.21	55.63	40.47
38	393.49	21.94	47.67	35.26	29.31	87.32	55.78	40.68
39	401.63	22.70	48.70	36.05	30.01	87.44	55.95	40.91
40	410.03	23.50	49.78	36.89	30.75	87.58	56.14	41.18
41	418.69	24.36	50.89	37.76	31.52	87.73	56.36	41.47
42	427.62	25.26	52.05	38.67	32.34	87.91	56.61	41.81
43	436.81	26.23	53.26	39.62	33.20	88.10	56.88	42.18
44	446.28	27.26	54.51	40.62	34.11	88.33	57.20	42.61
45	456.00	28.35	55.82	41.66	35.07	88.58	57.55	43.08
46	466.00	29.51	57.18	42.77	36.08	88.88	57.95	43.61
47	476.26	30.75	58.59	43.92	37.16	89.21	58.41	44.21
48	486.77	32.07	60.07	45.14	38.31	89.58	58.92	44.88
49	497.52	33.48	61.60	46.42	39.53	90.00	59.49	45.63
50	508.49	34.99	63.20	47.77	40.82	90.48	60.13	46.46
51	519.67	36.59	64.87	49.19	42.20	91.01	60.84	47.39
52	531.04	38.29	66.60	50.69	43.67	91.60	61.63	48.41
53	542.58	40.11	68.41	52.27	45.23	92.26	62.52	49.55
54	554.30	42.06	70.29	53.94	46.91	93.00	63.50	50.81
55	566.15	44.13	72.26	55.71	48.70	93.82	64.59	52.21
56	578.13	46.34	74.32	57.60	50.63	94.73	65.81	53.75
57	590.22	48.71	76.47	59.60	52.69	95.74	67.16	55.45
58	602.39	51.23	78.72	61.73	54.90	96.87	68.65	57.32
59	614.63	53.94	81.09	64.00	57.28	98.12	70.31	59.38
60	626.92	56.83	83.59	66.43	59.85	99.51	72.15	61.65
61	639.24	59.92	86.22	69.04	62.61	101.06	74.18	64.13
62	651.55	63.23	89.00	71.83	65.60	102.78	76.43	66.86
63	663.83	66.78	91.94	74.83	68.82	104.68	78.90	69.85
64	676.07	70.58	95.07	78.05	72.30	106.80	81.63	73.13
65	688.24	74.65	98.39	81.52	76.07	109.14	84.63	76.72
66	700.30	79.02	101.92	85.26	80.15	111.73	87.93	
67	712.23	83.70	105.69	89.29	84.57	114.58	91.55	
68	724.01	88.71	109.70	93.65	89.35	117.70	95.53	
69	735.60	94.08	113.98	98.36	94.52	121.13	99.90	
70	746.98	99.84	118.54	103.45	100.11	124.87	104.68	

*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death, American Experience, Three and One-Half per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	6.19	12.60	19.24	26.11	33.23	40.61	48.24
21	6.45	13.13	20.04	27.21	34.63	42.31	50.26
22	6.72	13.68	20.89	28.36	36.09	44.09	52.38
23	7.01	14.27	21.79	29.57	37.62	45.97	54.59
24	7.31	14.88	22.72	30.83	39.23	47.92	56.91
25	7.63	15.52	23.70	32.16	40.91	49.97	59.35
26	7.96	16.19	24.72	33.54	42.67	52.12	61.89
27	8.30	16.90	25.79	34.99	44.51	54.36	64.54
28	8.67	17.63	26.91	36.52	46.45	56.71	67.32
29	9.04	18.40	28.09	38.11	48.46	59.16	70.23
30	9.45	19.22	29.33	39.78	50.58	61.74	73.27
31	9.87	20.08	30.62	41.52	52.80	64.44	76.46
32	10.31	20.96	31.97	43.36	55.11	67.26	79.78
33	10.76	21.89	33.39	45.27	57.54	70.19	83.25
34	11.25	22.88	34.89	47.29	60.08	73.27	86.37
35	11.76	23.91	36.45	49.39	62.73	76.49	90.67
36	12.29	24.98	38.07	51.58	65.50	79.84	94.62
37	12.85	26.10	39.78	53.87	68.40	83.36	98.76
38	13.43	27.28	41.55	56.27	71.43	87.03	103.07
39	14.04	28.51	43.43	58.79	74.61	90.87	107.58
40	14.68	29.80	45.39	61.43	77.92	94.87	112.25
41	15.36	31.17	47.45	64.19	81.39	99.03	117.11
42	16.06	32.60	49.59	67.06	84.98	103.34	122.12
43	16.81	34.08	51.84	70.04	88.70	107.79	127.28
44	17.57	35.63	54.15	73.13	92.54	112.36	132.54
45	18.38	37.23	56.55	76.32	96.48	117.03	137.93
46	19.20	38.89	59.02	79.57	100.50	121.79	143.41
47	20.07	40.60	61.54	82.89	104.59	126.64	149.00
48	20.95	42.33	64.10	86.26	108.76	131.57	154.67
49	21.84	44.08	66.71	89.69	112.99	136.58	160.42
50	22.74	45.87	69.37	93.19	117.31	141.68	166.27
51	23.67	47.71	72.09	96.77	121.71	146.87	172.22
52	24.62	49.59	74.87	100.42	126.19	152.15	178.26
53	25.00	51.52	77.71	104.13	130.74	157.52	184.38
54	26.59	53.48	80.59	107.91	135.38	162.95	190.58
55	27.62	55.47	83.53	111.76	140.08	168.46	196.84
56	28.65	57.51	86.53	115.66	144.85	174.03	203.15
57	29.71	59.59	89.58	119.63	149.67	179.65	209.51
58	30.80	61.71	92.67	123.63	154.53	185.31	215.91
59	31.89	63.84	95.78	127.66	159.42	190.99	222.29
60	33.00	66.00	98.93	131.73	164.34	196.67	228.66
61	34.12	68.17	102.10	135.82	169.26	202.33	234.98
62	35.26	70.38	105.30	139.91	174.16	207.96	241.22
63	36.41	72.60	108.48	143.98	179.01	213.49	247.36
64	37.56	74.80	111.63	147.99	183.77	218.92	253.34
65	38.69	76.96	114.74	151.92	188.44	224.20	259.19
66	39.82	79.11	117.79	155.78	192.98	229.37	264.99
67	40.93	81.21	120.77	159.52	197.42	234.51	270.88
68	42.00	83.25	123.65	163.17	201.84	239.77	277.06
69	43.06	85.23	126.48	166.85	206.44	245.37	283.74
70	44.07	87.18	129.37	170.74	211.41	251.51	291.12

*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death, American Experience, Three and One-Half per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	56.14	64.32	72.78	81.54	90.60	99.98	109.66
21	58.49	67.00	75.82	84.94	94.37	104.12	114.19
22	60.95	69.82	79.00	88.49	98.30	108.44	118.93
23	63.52	72.76	82.32	92.20	102.41	112.97	123.87
24	66.22	75.85	85.79	96.08	106.71	117.69	129.02
25	69.04	79.06	89.42	100.13	111.19	122.61	134.39
26	71.98	82.42	93.21	104.36	115.87	127.74	139.98
27	75.06	85.94	97.17	108.77	120.74	133.09	145.81
28	78.29	89.62	101.31	113.38	125.83	138.66	151.88
29	81.66	93.46	105.63	118.18	131.13	144.47	158.21
30	85.18	97.46	110.14	123.20	136.66	150.52	164.80
31	88.86	101.65	114.84	128.43	142.42	156.84	171.65
32	92.70	106.01	119.74	133.88	148.43	163.39	178.77
33	96.70	110.57	124.86	139.56	154.68	170.22	186.15
34	100.90	115.34	130.20	145.48	161.19	177.30	193.81
35	105.27	120.31	135.76	151.65	167.94	184.64	201.72
36	109.84	125.48	141.55	158.04	174.93	192.22	209.85
37	114.60	130.87	147.56	164.67	182.17	200.02	218.22
38	119.56	136.47	153.79	171.52	189.61	208.04	226.78
39	124.71	142.28	160.25	178.58	197.26	216.26	235.56
40	130.06	148.29	166.89	185.83	205.10	224.68	244.52
41	135.60	154.48	173.71	193.26	213.13	233.27	253.66
42	141.29	160.82	180.68	200.86	221.32	242.02	262.96
43	147.12	167.31	187.81	208.61	229.65	250.93	272.39
44	153.08	173.93	195.08	216.49	238.12	259.95	281.93
45	159.16	180.68	202.47	224.50	246.71	269.09	291.60
46	165.34	187.54	209.98	232.61	255.41	278.34	301.35
47	171.63	194.51	217.58	240.83	264.21	287.67	311.18
48	178.01	201.56	225.28	249.14	273.09	297.08	321.06
49	184.48	208.71	233.07	257.53	282.04	306.53	330.98
50	191.04	215.96	240.96	266.01	291.05	316.05	340.95
51	197.71	223.30	248.93	274.56	300.13	325.61	350.94
52	204.47	230.72	256.97	283.16	309.26	335.21	360.93
53	211.30	238.21	265.07	291.83	318.42	344.79	370.88
54	218.20	245.76	273.22	300.52	327.58	354.35	380.78
55	225.15	253.36	281.41	309.21	336.71	363.86	390.58
56	232.16	261.00	289.59	317.88	345.79	373.27	400.25
57	239.20	268.64	297.76	326.50	354.78	382.57	409.78
58	246.24	276.26	305.88	335.03	363.66	391.70	419.13
59	253.26	283.82	313.90	343.44	372.37	400.68	428.38
60	260.23	291.30	321.81	351.70	380.93	409.55	437.60
61	267.11	298.67	329.57	359.81	389.40	418.41	446.94
62	273.89	305.89	337.19	367.82	397.86	427.40	456.51
63	280.53	312.97	344.72	375.86	406.48	436.65	466.46
64	287.01	319.96	352.28	384.05	415.37	446.30	476.87
65	293.43	327.00	360.01	392.55	424.70	456.46	487.81
66	299.91	334.26	368.11	401.54	434.58	467.20	499.39
67	306.65	341.90	376.73	411.14	445.10	478.63	511.42
68	313.82	350.13	386.01	421.43	456.38	490.57	524.07
69	321.64	359.09	396.06	432.55	468.24	503.21	537.60
70	330.25	368.89	407.01	444.31	480.86	516.79	552.49



*Terminal Net Values per \$1,000 of Whole Life Policies by Equa. Annual Premiums Till Death, American Experience, Three and One-Half per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	119.68	130.03	140.72	151.77	163.15	174.90
21	124.61	135.37	146.48	157.94	169.76	181.94
22	129.76	140.94	152.47	164.37	176.63	189.28
23	135.13	146.74	158.72	171.06	183.79	196.90
24	140.72	152.78	165.21	178.03	191.23	204.82
25	146.54	159.07	171.98	185.28	198.97	213.04
26	152.60	165.61	179.02	192.81	206.99	221.57
27	158.92	172.43	186.34	200.63	215.33	230.40
28	165.50	179.53	193.94	208.76	223.95	239.53
29	172.35	186.90	201.84	217.17	232.88	248.95
30	179.47	194.56	210.02	225.88	242.09	258.64
31	186.88	202.49	218.50	234.87	251.57	268.59
32	194.54	210.71	227.24	244.11	261.30	278.79
33	202.49	219.19	236.24	253.61	271.28	289.22
34	210.70	227.93	245.49	263.35	281.49	299.88
35	219.15	236.91	254.97	273.31	291.92	310.75
36	227.82	246.10	264.66	283.49	302.54	321.80
37	236.72	255.52	274.57	293.87	313.37	333.04
38	245.82	265.13	284.68	304.43	324.36	344.43
39	255.13	274.94	294.96	315.16	335.51	355.97
40	264.62	284.92	305.41	326.04	346.80	367.63
41	274.27	295.06	316.01	337.07	358.21	379.39
42	284.07	305.34	326.73	348.20	369.72	391.22
43	294.00	315.74	337.57	359.43	381.29	403.10
44	304.05	326.24	348.48	370.71	392.90	415.00
45	314.19	336.83	359.46	382.04	404.54	426.90
46	324.41	347.46	370.47	393.39	416.17	438.76
47	334.68	358.14	381.51	404.74	427.77	450.55
48	345.00	368.84	392.55	416.05	439.30	462.25
49	355.34	379.55	403.56	427.30	450.74	473.81
50	365.70	390.24	414.52	438.48	462.07	485.23
51	376.05	400.90	425.42	449.55	473.25	496.46
52	386.37	411.49	436.20	460.48	484.26	507.51
53	396.63	421.97	446.86	471.24	495.08	518.42
54	406.78	432.32	457.34	481.81	505.76	529.25
55	416.82	442.52	467.66	492.26	516.39	540.11
56	426.68	452.54	477.84	502.65	527.05	551.10
57	436.39	462.44	487.98	513.10	537.86	562.31
58	445.98	472.31	498.19	523.71	548.91	573.81
59	455.54	482.25	508.57	534.58	560.27	585.64
60	465.19	492.38	519.24	545.78	571.99	597.84
61	475.06	502.84	530.28	557.38	584.12	610.28
62	485.27	513.69	541.74	569.43	596.51	623.04
63	495.92	525.00	553.69	581.76	609.27	636.32
64	507.05	536.83	565.96	594.51	622.58	650.46
65	518.75	549.02	578.68	607.85	636.82	665.71
66	530.87	561.72	592.07	622.20	652.26	681.96
67	543.55	575.15	606.53	637.84	668.77	698.77
68	557.02	589.74	622.39	654.63	685.92	715.54
69	571.76	605.83	639.49	672.15	703.07	732.21
70	588.10	623.27	657.39	689.71	720.17	749.56



*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, American Experience, Three and One-Half per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	13.75	28.05	42.91	58.36	74.44	91.16	108.55
21	14.05	28.65	43.84	59.64	76.07	93.15	110.92
22	14.36	29.29	44.81	60.96	77.75	95.21	113.37
23	14.68	29.95	45.83	62.33	79.50	97.34	115.90
24	15.02	30.64	46.88	63.75	81.30	99.55	118.51
25	15.37	31.35	47.96	65.22	83.17	101.82	121.22
26	15.73	32.08	49.07	66.73	85.09	104.18	124.01
27	16.10	32.84	50.23	68.30	87.09	106.61	126.90
28	16.49	33.62	51.42	69.92	89.15	109.12	129.87
29	16.88	34.43	52.66	71.60	91.27	111.71	132.95
30	17.30	35.27	53.94	73.32	93.46	114.39	136.12
31	17.73	36.14	55.25	75.11	95.73	117.15	139.38
32	18.17	37.03	56.61	76.95	98.06	119.99	142.74
33	18.62	37.94	58.02	78.85	100.47	122.91	146.20
34	19.09	38.91	59.47	80.82	102.95	125.93	149.76
35	19.58	39.90	60.97	82.83	105.51	129.03	153.42
36	20.08	40.91	62.51	84.91	108.13	132.22	157.19
37	20.60	41.96	64.10	87.05	110.84	135.50	161.07
38	21.13	43.03	65.73	89.25	113.63	138.88	165.04
39	21.69	44.15	67.42	91.53	116.51	142.36	169.13
40	22.25	45.30	69.17	93.88	119.46	145.93	173.31
41	22.85	46.50	70.98	96.30	122.50	149.59	177.58
42	23.46	47.73	72.83	98.79	125.61	153.31	181.91
43	24.10	49.00	74.74	101.33	128.78	157.10	186.29
44	24.74	50.30	76.68	103.92	131.99	160.91	190.69
45	25.41	51.63	78.67	106.54	135.23	164.74	195.10
46	26.09	52.98	80.68	109.17	138.47	168.57	199.49
47	26.78	54.35	82.69	111.81	141.70	172.39	203.88
48	27.48	55.72	84.70	114.44	144.93	176.20	208.24
49	28.17	57.07	86.70	117.05	148.14	179.97	212.56
50	28.87	58.43	88.70	119.68	151.35	183.74	216.84
51	29.56	59.80	90.72	122.30	154.56	187.48	221.10
52	30.27	61.19	92.74	124.93	157.75	191.21	225.33
53	30.99	62.58	94.78	127.56	160.94	194.93	229.52
54	31.71	63.98	96.82	130.20	164.14	198.62	233.67
55	32.44	65.40	98.87	132.85	167.32	202.29	237.76
56	33.17	66.82	100.93	135.49	170.49	205.92	241.79
57	33.92	68.27	103.01	138.14	173.64	209.51	245.78
58	34.68	69.72	105.10	140.78	176.77	213.07	249.69
59	35.44	71.18	107.18	143.41	179.88	216.58	253.49
60	36.22	72.64	109.26	146.04	182.97	220.01	257.19
61	36.99	74.12	111.35	148.67	186.01	223.38	260.78
62	37.78	75.62	113.47	151.27	189.01	226.67	264.24
63	38.59	77.14	115.58	153.87	191.98	229.87	267.57
64	39.41	78.65	117.68	156.43	194.87	232.99	270.77
65	40.22	80.16	119.76	158.95	197.71	236.00	273.86
66	41.04	81.68	121.83	161.45	200.50	238.98	276.99
67	41.88	83.19	123.90	163.93	203.28	242.04	280.34
68	42.69	84.70	125.96	166.44	206.21	245.40	284.16
69	43.54	86.24	128.09	169.13	209.50	249.33	288.76
70	44.38	87.82	130.40	172.21	213.40	254.09	294.41

*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, American Experience, Three and One-Half per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	126.64	145.46	165.03	185.39	206.58	228.62	251.54
21	129.40	148.61	168.60	189.40	211.03	233.53	256.94
22	132.24	151.88	172.30	193.54	215.63	238.61	262.51
23	135.19	155.26	176.12	197.82	220.38	243.86	268.27
24	138.24	158.75	180.07	202.24	225.29	249.27	274.20
25	141.39	162.35	184.14	206.80	230.35	254.85	280.31
26	144.63	166.06	188.34	211.50	235.57	260.59	286.60
27	147.98	169.90	192.67	216.35	240.94	266.51	293.08
28	151.44	173.85	197.14	221.33	246.47	272.59	299.74
29	155.01	177.93	201.74	226.47	252.16	278.85	306.59
30	158.69	182.12	206.47	231.75	258.01	285.29	313.63
31	162.47	186.44	211.33	237.18	264.02	291.90	320.85
32	166.36	190.88	216.34	242.76	270.20	298.68	328.26
33	170.36	195.45	221.48	248.50	276.53	305.63	335.82
34	174.49	200.15	226.77	254.38	283.03	312.74	343.56
35	178.73	204.98	232.19	260.41	289.67	320.00	351.44
36	183.08	209.92	237.75	266.57	296.44	327.39	359.44
37	187.55	215.00	243.42	272.86	303.35	334.89	367.54
38	192.14	220.19	249.22	279.27	310.34	342.48	375.73
39	196.83	225.49	255.13	285.76	317.42	350.14	383.98
40	201.62	230.88	261.10	292.31	324.55	357.85	392.27
41	206.49	236.34	267.13	298.92	331.72	365.59	400.59
42	211.41	241.84	273.20	305.54	338.91	373.34	408.90
43	216.37	247.36	279.28	312.18	346.09	381.07	417.18
44	221.34	252.89	285.36	318.80	353.25	388.75	425.40
45	226.31	258.41	291.42	325.39	360.35	396.37	433.55
46	231.27	263.90	297.44	331.91	367.37	403.90	441.58
47	236.19	269.35	303.39	338.35	374.31	411.32	449.48
48	241.08	274.74	309.27	344.71	381.12	418.59	457.21
49	245.91	280.06	315.06	350.96	387.80	425.70	464.76
50	250.70	285.33	320.77	357.08	394.34	432.64	472.11
51	255.44	290.51	326.38	363.08	400.71	439.38	479.23
52	260.12	295.62	331.86	368.93	406.90	445.91	486.09
53	264.75	300.63	337.23	374.62	412.90	452.19	492.66
54	269.29	305.54	342.47	380.15	418.68	458.20	498.91
55	273.77	310.35	347.56	385.47	424.20	463.91	504.80
56	278.15	315.03	352.46	390.56	429.44	469.27	510.29
57	282.45	319.56	357.18	395.41	434.36	474.25	515.33
58	286.62	323.93	361.67	399.95	438.92	478.79	519.89
59	290.65	328.10	365.89	404.17	443.08	482.88	523.97
60	294.52	332.04	369.84	408.03	446.82	486.54	527.64
61	298.22	335.76	373.47	411.53	450.20	489.86	530.99
62	301.74	339.23	376.83	414.76	453.34	492.97	534.16
63	305.08	342.50	380.00	417.85	456.39	496.03	537.27
64	308.27	345.65	383.11	420.96	459.52	499.20	540.47
65	311.41	348.84	386.38	424.31	462.94	502.64	543.86
66	314.69	352.29	390.02	428.11	466.84	506.51	547.60
67	318.35	356.27	394.29	432.60	471.40	511.01	551.65
68	322.66	361.05	399.46	438.02	476.91	516.18	556.36
69	327.93	366.93	405.80	444.68	483.44	522.44	562.26
70	334.41	374.11	413.57	452.58	491.36	530.31	570.15

*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, American Experience, Three and One-Half per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	275.39	300.22	326.05	352.93	380.91	410.03
21	281.29	306.63	333.00	360.43	388.98	418.69
22	287.38	313.25	340.15	368.15	397.29	427.62
23	293.66	320.06	347.53	376.11	405.86	436.81
24	300.13	327.09	355.13	384.31	414.67	446.28
25	306.79	334.32	362.96	392.75	423.75	456.00
26	313.65	341.77	371.01	401.43	433.08	466.00
27	320.70	349.42	379.29	410.35	442.65	476.26
28	327.96	357.30	387.79	419.50	452.47	486.77
29	335.42	365.38	396.52	428.88	462.53	497.52
30	343.07	373.67	405.45	438.48	472.81	508.49
31	350.92	382.15	414.59	448.28	483.29	519.67
32	358.95	390.82	423.91	458.27	493.95	531.04
33	367.16	399.67	433.40	468.42	504.78	542.58
34	375.52	408.67	443.04	478.72	515.77	554.20
35	384.02	417.79	452.81	489.15	526.90	566.15
36	392.64	427.03	462.69	499.70	538.14	578.13
37	401.35	436.37	472.67	510.34	549.49	590.22
38	410.14	445.78	482.72	521.06	560.90	602.39
39	418.99	455.24	492.82	531.82	572.38	614.63
40	427.87	464.74	502.94	542.61	583.89	626.92
41	436.77	474.23	513.07	553.41	595.40	639.24
42	445.66	483.71	523.17	564.18	606.90	651.55
43	454.50	493.14	533.22	574.89	618.35	663.83
44	463.28	502.49	543.18	585.52	629.73	676.07
45	471.96	511.74	553.03	596.04	641.01	688.24
46	480.51	520.84	562.73	606.41	652.15	700.30
47	488.91	529.77	572.26	616.61	663.14	712.23
48	497.13	538.51	581.58	626.60	673.92	724.01
49	505.14	547.02	590.65	636.34	684.48	735.60
50	512.92	555.27	599.45	645.80	694.78	746.98
51	520.43	563.23	607.94	654.95	704.79	758.13
52	527.65	570.86	616.07	663.74	714.47	769.04
53	534.54	578.13	623.83	672.15	723.81	779.72
54	541.06	584.99	631.15	680.15	732.81	790.18
55	547.18	591.40	638.02	687.73	741.48	800.48
56	552.83	597.33	644.41	694.90	749.85	810.62
57	557.99	602.76	650.36	701.69	757.93	820.64
58	562.66	607.73	655.87	708.10	765.74	830.54
59	566.87	612.26	660.98	714.16	773.26	840.32
60	570.69	616.41	665.73	719.86	780.47	849.97
61	574.20	620.25	670.13	725.18	787.35	859.40
62	577.52	623.84	674.21	730.15	793.81	868.65
63	580.74	627.25	678.02	734.64	799.83	877.74
64	583.95	630.55	681.43	738.67	805.44	886.77
65	587.26	633.61	684.48	742.25	810.70	895.78
66	590.57	636.55	687.28	745.55	815.63	904.68
67	594.11	639.61	690.18	748.76	820.12	913.32
68	598.29	643.39	693.63	751.95	823.97	921.49
69	603.88	648.57	697.95	755.15	826.97	929.20
70	611.63	655.52	703.23	758.19	829.08	936.64



*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three and One-Half per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	32.71	66.79	102.29	139.29	177.84	218.02	259.90
21	32.71	66.78	102.28	139.27	177.82	217.99	259.86
22	32.71	66.77	102.27	139.26	177.80	217.97	259.83
23	32.70	66.77	102.27	139.25	177.79	217.95	259.80
24	32.71	66.78	102.27	139.25	177.78	217.93	259.78
25	32.71	66.78	102.27	139.25	177.78	217.92	259.76
26	32.71	66.78	102.28	139.25	177.77	217.91	259.74
27	32.72	66.79	102.28	139.26	177.78	217.91	259.73
28	32.73	66.80	102.30	139.27	177.79	217.91	259.72
29	32.73	66.81	102.32	139.29	177.80	217.92	259.73
30	32.74	66.84	102.35	139.32	177.83	217.95	259.74
31	32.76	66.87	102.38	139.35	177.87	217.98	259.77
32	32.78	66.90	102.42	139.41	177.92	218.03	259.80
33	32.79	66.93	102.47	139.47	177.98	218.09	259.85
34	32.82	66.99	102.54	139.55	178.07	218.17	259.92
35	32.86	67.06	102.63	139.65	178.18	218.28	260.03
36	32.90	67.13	102.73	139.77	178.30	218.41	260.16
37	32.95	67.22	102.85	139.91	178.47	218.59	260.34
38	33.00	67.32	102.99	140.09	178.67	218.81	260.56
39	33.08	67.44	103.17	140.31	178.93	219.08	260.84
40	33.15	67.59	103.38	140.58	179.23	219.41	261.16
41	33.25	67.78	103.65	140.89	179.59	219.78	261.53
42	33.36	67.99	103.94	141.26	179.99	220.20	261.95
43	33.50	68.23	104.28	141.67	180.45	220.67	262.38
44	33.63	68.51	104.65	142.12	180.94	221.16	262.84
45	33.80	68.81	105.06	142.60	181.45	221.66	263.29
46	33.97	69.13	105.50	143.10	181.97	222.17	263.76
47	34.17	69.48	105.95	143.62	182.51	222.70	264.24
48	34.37	69.84	106.42	144.15	183.07	223.25	264.72
49	34.57	70.19	106.89	144.69	183.65	223.80	265.21
50	34.79	70.58	107.39	145.28	184.26	224.39	265.73
51	35.01	70.98	107.94	145.90	184.92	225.02	266.29
52	35.26	71.43	108.53	146.58	185.63	225.71	266.90
53	35.53	71.91	109.17	147.32	186.40	226.47	267.58
54	35.82	72.44	109.87	148.13	187.26	227.30	268.32
55	36.13	73.00	110.62	149.01	188.19	228.21	269.12
56	36.47	73.61	111.44	149.96	189.19	229.19	270.00
57	36.84	74.29	112.34	151.00	190.29	230.25	270.96
58	37.25	75.01	113.30	152.11	191.46	231.40	271.98
59	37.67	75.78	114.32	153.28	192.71	232.62	273.05
60	38.13	76.60	115.41	154.56	194.05	233.91	274.17
61	38.61	77.48	116.58	155.91	195.46	235.26	275.34
62	39.13	78.42	117.84	157.35	196.96	236.69	276.55
63	39.70	79.44	119.17	158.88	198.54	238.17	277.81
64	40.30	80.50	120.57	160.48	200.19	239.73	279.12
65	40.92	81.61	122.03	162.13	201.91	241.35	280.51



*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three and One-Half per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.
20	303.55	349.05	396.50	445.97	497.56	551.38
21	303.50	349.00	396.43	445.90	497.49	551.29
22	303.46	348.95	396.38	445.83	497.41	551.20
23	303.43	348.91	396.32	445.76	497.33	551.12
24	303.40	348.87	396.27	445.70	497.25	551.03
25	303.37	348.82	396.21	445.63	497.17	550.94
26	303.34	348.78	396.16	445.56	497.09	550.84
27	303.31	348.75	396.11	445.50	497.00	550.73
28	303.30	348.72	396.06	445.43	496.91	550.63
29	303.29	348.69	396.01	445.36	496.82	550.52
30	303.29	348.67	395.98	445.29	496.74	550.41
31	303.30	348.66	395.94	445.24	496.65	550.30
32	303.32	348.66	395.92	445.19	496.58	550.20
33	303.35	348.68	395.91	445.17	496.52	550.10
34	303.42	348.72	395.94	445.16	496.48	550.02
35	303.51	348.80	395.99	445.17	496.45	549.94
36	303.64	348.90	396.07	445.20	496.43	549.87
37	303.80	349.05	396.17	445.26	496.43	549.79
38	304.01	349.23	396.31	445.34	496.43	549.70
39	304.27	349.45	396.48	445.43	496.42	549.59
40	304.57	349.71	396.66	445.52	496.40	549.46
41	304.92	349.99	396.85	445.60	496.37	549.31
42	305.28	350.28	397.04	445.67	496.31	549.11
43	305.66	350.57	397.22	445.73	496.22	548.87
44	306.04	350.86	397.39	445.75	496.09	548.57
45	306.42	351.14	397.54	445.74	495.91	548.21
46	306.81	351.40	397.66	445.69	495.67	547.79
47	307.19	351.66	397.75	445.60	495.38	547.29
48	307.57	351.90	397.82	445.48	495.04	546.72
49	307.95	352.13	397.87	445.31	494.63	546.06
50	308.36	352.38	397.92	445.12	494.18	545.33
51	308.80	352.65	397.96	444.90	493.67	544.53
52	309.28	352.93	398.01	444.67	493.13	543.67
53	309.80	353.25	398.07	444.43	492.55	542.73
54	310.38	353.61	398.15	444.18	491.93	541.71
55	311.02	354.01	398.25	443.91	491.25	540.58
56	311.72	354.45	398.35	443.60	490.48	539.33
57	312.48	354.92	398.44	443.25	489.61	537.91
58	313.27	355.39	398.50	442.80	488.60	536.28
59	314.09	355.86	398.49	442.24	487.40	534.42
60	314.93	356.29	398.42	441.55	486.03	532.36
61	315.78	356.69	398.25	440.72	484.52	530.19
62	316.64	357.07	398.03	439.86	483.01	528.04
63	317.52	357.45	397.86	439.11	481.67	526.10
64	318.46	357.95	397.89	438.65	480.68	524.55
65	319.55	358.73	398.32	438.69	480.27	523.57

*Terminal Net Values per \$1,000 of Twenty-Year Endowment Policies by Equal Annual Premiums Till Maturity, American Experience, Three and One-Half per Cent.*

AGE.	14th Year.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.
20	607.52	666.10	727.23	791.05	857.69	927.28
21	607.43	666.00	727.14	790.97	857.62	927.24
22	607.34	665.91	727.05	790.88	857.54	927.20
23	607.24	665.81	726.94	790.78	857.47	927.15
24	607.14	665.70	726.84	790.68	857.38	927.10
25	607.04	665.59	726.72	790.57	857.29	927.04
26	606.92	665.46	726.60	790.45	857.19	926.98
27	606.80	665.34	726.46	790.33	857.09	926.91
28	606.68	665.20	726.32	790.19	856.97	926.84
29	606.55	665.06	726.17	790.05	856.85	926.76
30	606.42	664.91	726.02	789.89	856.71	926.67
31	606.29	664.76	725.85	789.73	856.57	926.58
32	606.16	664.60	725.68	789.55	856.40	926.47
33	606.03	664.44	725.49	789.35	856.23	926.35
34	605.91	664.28	725.29	789.14	856.03	926.22
35	605.78	664.10	725.07	788.89	855.81	926.07
36	605.64	663.89	724.82	788.62	855.56	925.90
37	605.48	663.66	724.54	788.32	855.28	925.72
38	605.30	663.40	724.22	787.98	854.96	925.51
39	605.09	663.11	723.86	787.59	854.61	925.27
40	604.86	662.78	723.45	787.16	854.21	925.01
41	604.58	662.39	722.99	786.67	853.77	924.71
42	604.25	661.94	722.46	786.11	853.26	924.38
43	603.86	661.43	721.86	785.48	852.70	924.00
44	603.40	660.84	721.17	784.76	852.05	923.58
45	602.88	660.17	720.40	783.96	851.33	923.10
46	602.27	659.39	719.51	783.05	850.52	922.57
47	601.57	658.52	718.52	782.03	849.61	921.97
48	600.77	657.53	717.41	780.89	848.59	921.30
49	599.87	656.43	716.16	779.61	847.46	920.56
50	598.89	655.22	714.79	778.20	846.20	919.72
51	597.81	653.88	713.28	776.64	844.80	918.80
52	596.62	652.41	711.61	774.92	843.25	917.77
53	595.32	650.80	709.78	773.02	841.54	916.63
54	593.91	649.03	707.75	770.91	839.63	915.37
55	592.34	647.08	705.50	768.57	837.52	913.98
56	590.59	644.88	702.99	765.97	835.19	912.43
57	588.63	642.42	700.19	763.09	832.63	910.73
58	586.39	639.68	697.11	759.95	829.81	908.86
59	583.91	636.68	693.77	756.54	826.75	906.80
60	581.22	633.47	690.22	752.90	823.44	904.54
61	578.43	630.15	686.51	749.05	819.88	902.05
62	575.68	626.85	682.76	745.05	816.10	899.32
63	573.13	623.69	679.04	740.98	812.06	896.33
64	570.94	620.80	675.46	736.76	807.74	893.06
65	569.25	618.29	671.93	732.41	803.13	889.47

*Valuation Columns.*Am. Exp.  $3\frac{1}{2}$  %.

$$u_x = \frac{D_x}{D_{x+1}}$$

$$k_x = \frac{C_x}{D_{x+1}}$$

AGE.	$u_x$	$k_x$	AGE.	$u_x$	$k_x$
20	1.043 141	0.007 866	60	1.063 385	0.027 425
21	1.043 195	0.007 917	61	1.065 780	0.029 739
22	1.043 248	0.007 969	62	1.068 433	0.032 303
23	1.043 303	0.008 022	63	1.071 365	0.035 136
24	1.043 358	0.008 076	64	1.074 625	0.038 285
25	1.043 415	0.008 130	65	1.078 270	0.041 807
26	1.043 484	0.008 197	66	1.082 304	0.045 704
27	1.043 554	0.008 264	67	1.086 782	0.050 031
28	1.043 625	0.008 333	68	1.091 774	0.054 855
29	1.043 710	0.008 415	69	1.097 284	0.060 178
30	1.043 796	0.008 498	70	1.103 403	0.066 090
31	1.043 884	0.008 583	71	1.110 117	0.072 576
32	1.043 986	0.008 682	72	1.117 388	0.079 602
33	1.044 102	0.008 795	73	1.125 218	0.087 167
34	1.044 221	0.008 910	74	1.133 660	0.095 323
35	1.044 343	0.009 027	75	1.142 852	0.104 204
36	1.044 493	0.009 172	76	1.152 960	0.113 971
37	1.044 647	0.009 320	77	1.164 314	0.124 941
38	1.044 830	0.009 498	78	1.177 243	0.137 433
39	1.045 018	0.009 679	79	1.192 031	0.151 720
40	1.045 238	0.009 891	80	1.209 771	0.168 861
41	1.045 463	0.010 109	81	1.230 099	0.188 502
42	1.045 721	0.010 359	82	1.253 477	0.211 089
43	1.046 001	0.010 629	83	1.280 245	0.236 952
44	1.046 331	0.010 947	84	1.312 384	0.268 004
45	1.046 684	0.011 289	85	1.353 917	0.308 133
46	1.047 106	0.011 697	86	1.409 469	0.361 806
47	1.047 571	0.012 146	87	1.484 979	0.434 762
48	1.048 111	0.012 668	88	1.584 244	0.530 671
49	1.048 745	0.013 280	89	1.713 188	0.655 254
50	1.049 463	0.013 974	90	1.897 500	0.833 333
51	1.050 272	0.014 755	91	2.213 750	1.138 889
52	1.051 177	0.015 629	92	2.829 873	1.734 177
53	1.052 185	0.016 604	93	3.893 571	2.761 905
54	1.053 323	0.017 704	94	7.245 000	6.000 000
55	1.054 585	0.018 922			
56	1.055 999	0.020 289			
57	1.057 563	0.021 800			
58	1.059 296	0.023 474			
59	1.061 234	0.025 347			



*Commutation Columns—Combined Experience, Four per Cent.*

AGE.	$D_x$	$N_x$	$M_x$	$R_x$
10	67 556.41688	1 381 771.33883	14 411.36539	427 355.11784
11	64 518.97645	1 314 214.92194	13 972.24868	412 943.75245
12	61 616.49894	1 249 695.94550	13 551.27027	398 971.50377
13	58 843.04781	1 188 079.44656	13 147.68448	385 420.23350
14	56 192.36788	1 129 236.39875	12 760.19870	372 272.54902
15	53 658.54048	1 073 044.03086	12 387.61622	359 512.35032
16	51 236.49808	1 019 385.49038	12 029.36333	347 124.73410
17	48 920.87672	968 148.99230	11 684.37701	335 095.37027
18	46 707.09281	919 228.11558	11 352.16529	323 410.99325
19	44 590.28253	872 521.02277	11 031.78165	312 058.82797
20	42 566.29770	827 930.74024	10 722.80769	301 027.04631
21	40 630.72555	785 364.44255	10 424.40084	290 304.23862
22	38 779.80981	744 733.71699	10 136.20531	279 879.83779
23	37 009.95040	705 953.90718	9 857.87705	269 743.63248
24	35 317.30695	668 943.95678	9 588.69323	259 885.75543
25	33 698.61793	633 626.64983	9 328.36217	250 297.06220
26	32 150.75616	599 928.03190	9 076.60108	240 968.70003
27	30 670.37656	567 777.27575	8 832.78903	231 892.09895
28	29 254.64465	537 106.89918	8 596.68698	223 059.30996
29	27 900.52090	507 852.25454	8 367.74187	214 462.62293
30	26 605.43450	479 951.73364	8 145.75243	206 094.88106
31	25 366.62195	453 346.29915	7 930.22583	197 949.12862
32	24 181.75011	427 979.67720	7 720.99330	190 018.90280
33	23 048.30493	403 797.92708	7 517.61542	182 297.90950
34	21 964.16759	380 749.62216	7 319.95135	174 780.29408
35	20 927.30299	358 785.45457	7 127.86243	167 460.34272
36	19 935.51281	337 858.15158	6 940.96852	160 332.48029
37	18 986.94796	317 922.63877	6 759.15416	153 391.51178
38	18 079.83167	298 935.69081	6 582.30510	146 632.35761
39	17 212.24015	280 855.85914	6 410.09172	140 050.05251
40	16 382.55823	263 643.61899	6 242.41904	133 639.96079
41	15 589.23333	247 261.06076	6 079.19253	127 397.54175
42	14 830.58054	231 671.82743	5 920.12563	121 318.34923
43	14 104.81747	216 841.24690	5 764.76951	115 398.22359
44	13 409.73877	202 736.42943	5 612.18379	109 633.45408
45	12 743.15379	189 326.69066	5 461.35799	104 021.27029
46	12 103.39849	176 583.53687	5 311.72400	98 559.91230
47	11 488.46443	164 480.13838	5 162.30526	93 248.18830
48	10 897.29735	152 991.67395	5 013.00220	88 085.88304
49	10 328.75625	142 094.37660	4 863.58792	83 072.88084
50	9 781.91888	131 765.62035	4 714.01040	78 209.29292
51	9 255.77818	121 983.70147	4 564.09735	73 495.28252
52	8 749.39490	112 727.92330	4 413.70554	68 931.18517
53	8 261.89245	103 978.52840	4 262.71828	64 517.47963
54	7 792.45209	95 716.63595	4 111.04302	60 254.76135



*Commutation Columns—Combined Experience, Four per Cent.*

AGE.	D <sub>x</sub>	N <sub>x</sub>	M <sub>x</sub>	R <sub>x</sub>
55	7 340.53974	87 924.18386	3 958.84036	56 143.71833
56	6 905.30136	80 583.64411	3 805.93043	52 184.87797
57	6 486.16133	73 678.34276	3 652.37892	48 378.94754
58	6 082.77604	67 192.18143	3 498.46137	44 726.56862
59	5 694.49826	61 109.40538	3 344.13652	41 228.10724
60	5 320.81583	55 414.90712	3 189.47324	37 883.97073
61	4 960.96468	50 094.09129	3 034.26886	34 694.49748
62	4 614.59537	45 133.12661	2 878.70589	31 660.22862
63	4 281.27754	40 518.53124	2 722.87249	28 781.52273
64	3 960.84136	36 237.25370	2 567.10085	26 058.65024
65	3 653.01721	32 276.41233	2 411.61673	23 491.54940
66	3 357.67853	28 623.39512	2 256.77872	21 079.93266
67	3 074.81439	25 265.71659	2 103.05606	18 823.15394
68	2 804.36609	22 190.90220	1 950.86985	16 720.09788
69	2 546.49961	19 386.53611	1 800.86360	14 769.22804
70	2 301.43067	16 840.03651	1 653.73695	12 968.36444
71	2 069.22319	14 538.60584	1 510.04605	11 314.62748
72	1 850.04836	12 469.38265	1 370.45672	9 804.58144
73	1 644.04416	10 619.33428	1 235.60822	8 434.12471
74	1 451.36925	8 975.29013	1 106.16578	7 198.51649
75	1 272.08636	7 523.92088	982.70479	6 092.35071
76	1 106.27459	6 251.83452	865.81942	5 109.64592
77	953.97107	5 145.55993	756.06492	4 243.82651
78	815.03142	4 191.58886	653.81646	3 487.76159
79	689.29364	3 376.55745	559.42605	2 833.94513
80	576.57769	2 687.26380	473.22139	2 274.51908
81	476.56014	2 110.68611	395.37990	1 801.29768
82	388.83844	1 634.12597	325.98744	1 405.91778
83	312.86774	1 245.28753	264.97206	1 079.93034
84	247.91392	932.41979	212.05162	814.95828
85	193.16349	684.50587	166.83632	602.90666
86	147.64096	491.34241	128.74317	436.07034
87	110.37861	343.70145	97.15933	307.32717
88	80.42417	233.32284	71.45022	210.16784
89	56.81705	152.89866	50.93634	138.71763
90	38.65843	96.08161	34.96299	87.78129
91	25.13801	57.42318	22.92943	52.81830
92	15.44570	32.28516	14.20396	29.88887
93	8.83282	16.83946	8.18514	15.68491
94	4.60982	8.00665	4.30187	7.49976
95	2.14399	3.39683	2.01334	3.19789
96	.85704	1.25284	.80885	1.18455
97	.28954	.39580	.27432	.37569
98	.08566	.10625	.08158	.10138
99	.02059	.02059	.01980	.01980

*Net Premiums per \$1,000, Combined Experience, Four per Cent.*

AGE.	Single Premium.	Whole Life.	10 Pay- ment Life.	15 Pay- ment Life.	20 Pay- ment Life.	Endow- ment 10 Years.	Endow- ment 15 Years.	Endow- ment 20 Years.
20	251.91	12.95	30.81	22.86	19.00	83.86	52.27	36.97
21	256.56	13.27	31.40	23.29	19.37	83.91	52.33	37.05
22	261.38	13.61	32.00	23.75	19.76	83.97	52.40	37.12
23	266.36	13.96	32.63	24.22	20.15	84.03	52.47	37.21
24	271.50	14.33	33.27	24.71	20.57	84.09	52.54	37.29
25	276.82	14.72	33.94	25.21	21.00	84.15	52.62	37.39
26	282.31	15.13	34.64	25.74	21.44	84.22	52.70	37.48
27	287.99	15.56	35.35	26.28	21.90	84.29	52.79	37.59
28	293.86	16.01	36.09	26.84	22.38	84.37	52.88	37.70
29	299.91	16.48	36.86	27.43	22.88	84.45	52.98	37.82
30	306.17	16.97	37.66	28.03	23.39	84.54	53.08	37.95
31	312.62	17.49	38.48	28.65	23.93	84.63	53.19	38.09
32	319.29	18.04	39.33	29.30	24.49	84.72	53.31	38.25
33	326.17	18.62	40.21	29.97	25.07	84.82	53.44	38.41
34	333.27	19.23	41.12	30.67	25.68	84.92	53.57	38.60
35	340.60	19.87	42.06	31.40	26.32	85.03	53.72	38.80
36	348.17	20.54	43.04	32.15	26.98	85.15	53.89	39.03
37	355.99	21.26	44.05	32.94	27.67	85.28	54.07	39.28
38	364.07	22.02	45.10	33.76	28.40	85.42	54.28	39.56
39	372.41	22.82	46.20	34.62	29.17	85.58	54.51	39.87
40	381.04	23.68	47.33	35.53	29.98	85.76	54.77	40.21
41	389.96	24.59	48.53	36.47	30.83	85.98	55.07	40.61
42	399.18	25.55	49.77	37.47	31.74	86.22	55.41	41.04
43	408.71	26.58	51.08	38.52	32.69	86.51	55.79	41.53
44	418.52	27.68	52.44	39.63	33.71	86.84	56.22	42.08
45	428.57	28.85	53.86	40.78	34.77	87.21	56.70	42.68
46	438.86	30.08	55.33	41.99	35.90	87.62	57.23	43.34
47	449.35	31.39	56.85	43.25	37.08	88.06	57.80	44.06
48	460.02	32.77	58.43	44.57	38.32	88.55	58.43	44.85
49	470.88	34.23	60.05	45.95	39.63	89.08	59.11	45.71
50	481.91	35.78	61.74	47.38	41.02	89.66	59.86	46.65
51	493.11	37.42	63.49	48.89	42.48	90.29	60.68	47.68
52	504.46	39.15	65.30	50.46	44.02	90.98	61.58	48.81
53	515.95	41.00	67.17	52.12	45.66	91.73	62.56	50.03
54	527.57	42.95	69.12	53.86	47.39	92.55	63.63	51.37
55	539.31	45.03	71.14	55.69	49.24	93.45	64.80	52.84
56	551.16	47.23	73.25	57.63	51.20	94.43	66.09	
57	563.10	49.57	75.44	59.67	53.29	95.52	67.51	
58	575.14	52.07	77.75	61.84	55.53	96.71	69.06	
59	587.26	54.72	80.15	64.15	57.92	98.02	70.77	
60	599.43	57.56	82.68	66.60	60.49	99.47	72.64	
61	611.63	60.57	85.34	69.21	63.24	101.07		
62	623.83	63.78	88.13	71.99	66.18	102.81		
63	636.00	67.20	91.07	74.96	69.33	104.73		
64	648.12	70.84	94.16	78.12	72.71	106.83		
65	660.17	74.72	97.43	81.50	76.34	109.12		
66	672.12	78.85	100.88	85.12	80.22			
67	683.97	83.24	104.53	88.99	84.38			
68	695.65	87.91	108.39	93.14	88.85			
69	707.19	92.89	112.48	97.59	93.63			
70	718.57	98.20	116.85	102.36	98.77			

*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death. Combined Experience, Four Per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	6.22	12.66	19.31	26.19	33.30	40.64	48.23
21	6.47	13.17	20.09	27.24	34.64	42.27	50.16
22	6.74	13.71	20.90	28.34	36.03	43.97	52.17
23	7.01	14.26	21.75	29.49	37.48	45.74	54.26
24	7.30	14.84	22.64	30.69	39.00	47.59	56.45
25	7.60	15.45	23.56	31.94	40.58	49.51	58.73
26	7.91	16.08	24.52	33.24	42.23	51.52	61.11
27	8.24	16.75	25.53	34.60	43.96	53.62	63.59
28	8.58	17.43	26.58	36.02	45.76	55.81	66.20
29	8.93	18.16	27.68	37.50	47.64	58.12	68.93
30	9.31	18.91	28.83	39.06	49.63	60.54	71.80
31	9.70	19.70	30.03	40.70	51.71	63.08	74.84
32	10.10	20.54	31.31	42.43	53.91	65.78	78.04
33	10.54	21.42	32.65	44.25	56.25	68.63	81.43
34	11.00	22.35	34.07	46.20	58.71	71.65	85.03
35	11.48	23.34	35.59	48.25	61.34	74.86	88.84
36	11.99	24.39	37.19	50.43	64.11	78.26	92.87
37	12.55	25.51	38.90	52.75	67.08	81.87	97.09
38	13.12	26.69	40.72	55.22	70.20	85.62	101.43
39	13.74	27.96	42.65	57.83	73.46	89.48	105.88
40	14.41	29.31	44.70	60.55	76.79	93.42	110.36
41	15.12	30.73	46.81	63.29	80.16	97.35	114.85
42	15.85	32.18	48.91	66.04	83.49	101.26	119.32
43	16.58	33.59	51.00	68.73	86.78	105.13	123.80
44	17.30	34.99	53.02	71.38	90.04	109.02	128.28
45	18.01	36.36	55.04	74.03	93.34	112.94	132.80
46	18.69	37.71	57.05	76.72	96.67	116.90	137.38
47	19.39	39.10	59.14	79.47	100.09	120.95	142.05
48	20.10	40.54	61.28	82.30	103.57	125.09	146.83
49	20.86	42.02	63.47	85.19	107.14	129.34	151.73
50	21.62	43.52	65.70	88.13	110.79	133.66	156.72
51	22.39	45.06	67.98	91.14	114.52	138.09	161.84
52	23.19	46.63	70.33	94.24	118.34	142.64	167.09
53	24.00	48.26	72.74	97.42	122.29	147.32	172.47
54	24.85	49.94	75.22	100.70	126.35	152.12	177.93
55	25.72	51.65	77.78	104.08	130.51	156.98	183.46
56	26.61	53.43	80.42	107.55	134.72	161.90	189.01
57	27.56	55.29	83.15	111.07	138.99	166.84	194.59
58	28.52	57.18	85.88	114.59	143.23	171.77	200.14
59	29.50	59.05	88.60	118.08	147.46	176.66	205.63
60	30.45	60.90	91.28	121.54	151.63	181.49	211.02
61	31.41	62.74	93.96	124.99	155.78	186.25	216.35
62	32.35	64.58	96.62	128.41	159.86	190.94	221.61
63	33.31	66.41	99.27	131.77	163.89	195.59	226.85
64	34.25	68.23	101.86	135.09	167.87	200.21	232.02
65	35.19	70.01	104.41	138.37	171.84	204.79	237.17



*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death. Combined Experience, Four per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	56.07	64.17	72.53	81.16	90.07	99.26	108.75
21	58.31	66.72	75.41	84.37	93.62	103.17	113.04
22	60.64	69.38	78.41	87.72	97.33	107.26	117.51
23	63.07	72.15	81.53	91.20	101.20	111.52	122.17
24	65.60	75.04	84.78	94.85	105.24	115.97	127.07
25	68.24	78.06	88.20	98.67	109.47	120.65	132.19
26	71.00	81.22	91.76	102.65	113.92	125.54	137.56
27	73.89	84.52	95.50	106.85	118.57	130.69	143.22
28	76.92	87.98	99.43	111.25	123.46	136.10	149.16
29	80.09	91.64	103.56	115.88	128.62	141.80	155.40
30	83.45	95.48	107.91	120.77	134.06	147.79	161.93
31	86.98	99.53	112.51	125.93	139.79	154.05	168.68
32	90.72	103.82	117.37	131.36	145.77	160.54	175.66
33	94.67	108.36	122.50	137.05	151.97	167.24	182.80
34	98.86	113.15	127.86	142.94	158.38	174.10	190.11
35	103.29	118.16	133.41	149.02	164.92	181.11	197.57
36	107.92	123.35	139.13	155.22	171.60	188.25	205.18
37	112.71	128.69	144.97	161.54	178.39	195.53	212.92
38	117.61	134.10	150.89	167.95	185.32	202.92	220.76
39	122.59	139.60	156.89	174.47	192.32	210.40	228.71
40	127.60	145.14	162.97	181.06	199.40	217.96	236.73
41	132.64	150.73	169.09	187.69	206.53	225.57	244.82
42	137.69	156.33	175.22	194.35	213.68	233.23	252.95
43	142.74	161.94	181.37	201.02	220.87	240.92	261.11
44	147.80	167.56	187.54	207.73	228.11	248.65	269.36
45	152.91	173.24	193.79	214.53	235.43	256.50	277.70
46	158.08	179.02	200.13	221.41	242.86	264.45	286.15
47	163.37	184.90	206.59	228.45	250.45	272.56	294.71
48	168.78	190.90	213.19	235.63	258.18	280.76	303.35
49	174.30	197.06	219.95	242.96	266.01	289.07	312.06
50	179.95	203.34	226.84	250.38	273.92	297.41	320.81
51	185.74	209.76	233.82	257.88	281.89	305.81	329.58
52	191.66	216.27	240.88	265.44	289.91	314.23	338.36
53	197.66	222.86	248.00	273.05	297.95	322.65	347.10
54	203.75	229.51	255.18	280.68	306.00	331.04	355.79
55	209.87	236.19	262.35	288.31	313.99	339.37	364.42
56	216.02	242.87	269.52	295.88	321.93	347.63	372.98
57	222.18	249.55	276.63	303.39	329.80	355.84	381.47
58	228.28	256.13	283.65	310.81	337.59	363.94	389.84
59	234.30	262.63	290.58	318.14	345.27	371.93	398.09
60	240.21	269.02	297.42	325.37	352.84	379.80	406.22
61	246.06	275.36	304.18	332.51	360.32	387.58	414.26
62	251.86	281.62	310.87	339.58	367.72	395.26	422.19
63	257.60	287.83	317.50	346.58	375.05	402.87	430.07
64	263.29	293.98	324.07	353.51	382.30	410.44	437.87
65	268.95	300.10	330.59	360.40	389.53	417.94	445.58



*Terminal Net Values per \$1,000 of Whole Life Policies by Equal Annual Premiums Till Death. Combined Experience Four per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	118.56	128.68	139.12	149.93	161.08	172.61
21	123.22	133.73	144.62	155.83	167.43	179.43
22	128.09	139.03	150.33	162.01	174.08	186.57
23	133.19	144.56	156.32	168.48	181.05	194.03
24	138.52	150.36	162.61	175.27	188.34	201.80
25	144.12	156.45	169.20	182.38	195.94	209.84
26	149.99	162.84	176.12	189.78	203.79	218.13
27	156.17	169.55	183.32	197.44	211.89	226.62
28	162.65	176.53	190.78	205.35	220.20	235.31
29	169.41	183.78	198.47	213.45	228.70	244.20
30	176.42	191.25	206.36	221.75	237.39	253.29
31	183.65	198.90	214.44	230.22	246.28	262.57
32	191.06	206.74	222.69	238.90	255.35	272.02
33	198.65	214.75	231.13	247.75	264.59	281.64
34	206.39	222.94	239.74	256.76	273.99	291.42
35	214.30	231.28	248.50	265.92	283.54	301.35
36	222.36	239.77	257.40	275.22	293.23	311.42
37	230.54	248.38	266.42	284.66	303.06	321.60
38	238.83	257.10	275.56	294.20	312.98	331.91
39	247.22	265.93	284.82	303.84	323.03	342.33
40	255.70	274.85	294.14	313.59	333.17	352.84
41	264.25	283.82	303.56	323.42	343.37	363.37
42	272.83	292.87	313.03	333.30	353.59	373.90
43	281.47	301.96	322.55	343.18	363.81	384.39
44	290.19	311.13	332.10	353.08	374.01	394.86
45	299.01	320.35	341.69	362.99	384.21	405.30
46	307.89	329.62	351.31	372.92	394.39	415.71
47	316.86	338.96	360.98	382.86	404.58	426.07
48	325.89	348.34	370.66	392.81	414.72	436.37
49	334.98	357.75	380.36	402.72	424.81	446.62
50	344.07	367.16	389.99	412.56	434.83	456.79
51	353.18	376.52	399.58	422.34	444.79	466.88
52	362.24	385.83	409.11	432.07	454.66	476.87
53	371.25	395.09	418.59	441.72	464.45	486.76
54	380.21	404.29	427.99	451.28	474.14	496.55
55	389.11	413.41	437.30	460.74	483.72	506.21
56	397.92	422.44	446.50	470.08	493.17	515.79
57	406.65	431.37	455.60	479.31	502.50	525.16
58	415.26	440.17	464.56	488.40	511.71	534.43
59	423.74	448.84	473.39	497.37	520.76	543.52
60	432.09	457.38	482.10	506.20	529.65	552.49
61	440.34	465.83	490.69	514.88	538.44	561.38
62	448.51	474.18	499.15	523.47	547.16	570.31
63	456.60	482.40	507.54	532.02	555.95	579.44
64	464.57	490.57	515.90	540.65	564.95	588.90
65	472.50	498.73	524.36	549.52	574.33	598.93

*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty  
Equal Annual Premiums, Combined Experience, Four Per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	12.56	25.64	39.26	53.44	68.19	83.56	99.56
21	12.86	26.27	40.20	54.71	69.82	85.54	101.93
22	13.18	26.89	41.17	56.03	71.50	87.59	104.35
23	13.50	27.54	42.16	57.37	73.21	89.69	106.85
24	13.83	28.23	43.20	58.79	74.99	91.87	109.44
25	14.18	28.92	44.26	60.21	76.82	94.09	112.08
26	14.53	29.63	45.35	61.70	78.71	96.40	114.81
27	14.90	30.38	46.48	63.22	80.65	98.76	117.62
28	15.26	31.13	47.64	64.79	82.63	101.20	120.52
29	15.65	31.91	48.82	66.40	84.68	103.71	123.52
30	16.04	32.71	50.05	68.06	86.82	106.32	126.61
31	16.45	33.55	51.32	69.80	89.03	109.02	129.84
32	16.87	34.40	52.64	71.58	91.30	111.82	133.18
33	17.31	35.30	54.01	73.45	93.69	114.75	136.67
34	17.77	36.24	55.44	75.41	96.19	117.80	140.32
35	18.25	37.22	56.93	77.44	98.79	120.98	144.11
36	18.75	38.24	58.50	79.58	101.51	124.34	148.08
37	19.28	39.31	60.15	81.82	104.37	127.82	152.17
38	19.83	40.44	61.88	84.18	107.36	131.42	156.35
39	20.41	41.64	63.71	86.64	110.43	135.07	160.56
40	21.03	42.90	65.61	89.17	113.53	138.74	164.74
41	21.68	44.20	67.55	91.69	116.63	142.36	168.88
42	22.35	45.51	69.45	94.18	119.65	145.89	172.94
43	23.01	46.79	71.32	96.58	122.57	149.35	176.94
44	23.63	48.01	73.09	98.88	125.43	152.74	180.84
45	24.25	49.17	74.80	101.14	128.23	156.06	184.70
46	24.80	50.31	76.48	103.37	130.99	159.37	188.51
47	25.40	51.44	78.18	105.62	133.78	162.66	192.32
48	25.97	52.60	79.90	107.88	136.56	165.95	196.12
49	26.56	53.76	81.61	110.12	139.32	169.23	199.90
50	27.15	54.92	83.33	112.38	142.10	172.52	203.66
51	27.75	56.09	85.05	114.64	144.88	175.79	207.44
52	28.34	57.26	86.79	116.92	147.67	179.11	211.23
53	28.95	58.47	88.55	119.20	150.51	182.43	215.04
54	29.59	59.68	90.32	121.54	153.36	185.79	218.80
55	30.20	60.90	92.14	123.92	156.25	189.11	222.53
56	30.84	62.18	94.01	126.34	159.13	192.41	226.18
57	31.52	63.51	95.92	128.76	162.02	195.67	229.77
58	32.22	64.84	97.81	131.15	164.82	198.84	233.24
59	32.93	66.16	99.68	133.47	167.56	201.93	236.59
60	33.59	67.43	101.48	135.75	170.21	204.87	239.78

*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, Combined Experience, Four per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.	14th Year.
20	116.23	133.59	151.68	170.52	190.16	210.62	231.96
21	118.98	136.74	155.24	174.52	194.60	215.53	237.36
22	121.80	139.97	158.90	178.62	199.17	220.60	242.93
23	124.72	143.31	162.69	182.87	203.90	225.83	248.69
24	127.72	146.75	166.58	187.24	208.77	231.22	254.63
25	130.79	150.28	170.58	191.74	213.78	236.77	260.76
26	133.97	153.93	174.72	196.39	218.97	242.52	267.09
27	137.25	157.71	179.00	201.20	224.33	248.47	273.66
28	140.63	161.59	183.41	206.16	229.88	254.62	280.44
29	144.12	165.60	187.97	211.29	235.62	260.99	287.46
30	147.75	169.77	192.71	216.63	241.57	267.59	294.69
31	151.51	174.10	197.63	222.18	247.75	274.39	302.09
32	155.43	178.60	202.76	227.92	254.12	281.35	309.65
33	159.50	183.29	208.08	233.85	260.63	288.44	317.30
34	163.76	188.17	213.56	239.92	267.28	295.63	325.04
35	168.19	193.21	219.18	246.10	273.99	302.91	332.88
36	172.76	198.35	224.87	252.34	280.76	310.21	340.76
37	177.42	203.57	230.62	258.60	287.57	317.57	348.68
38	182.16	208.82	236.40	264.90	294.41	324.97	356.64
39	186.89	214.07	242.16	271.21	301.25	332.36	364.59
40	191.59	219.29	247.91	277.49	308.08	339.72	372.51
41	196.25	224.49	253.63	283.74	314.86	347.05	380.41
42	200.83	229.60	259.27	289.90	321.56	354.31	388.23
43	205.34	234.63	264.82	295.97	328.16	361.45	395.95
44	209.78	239.58	270.29	301.98	334.69	368.53	403.61
45	214.16	244.45	275.68	307.89	341.13	375.54	411.18
46	218.46	249.28	281.01	313.72	347.51	382.44	418.65
47	222.79	254.11	286.33	319.56	353.85	389.33	426.04
48	227.07	258.87	291.61	325.32	360.12	396.06	433.29
49	231.34	263.64	296.86	331.05	366.30	402.69	440.35
50	235.61	268.37	302.05	336.68	372.33	409.14	447.25
51	239.86	273.09	307.18	342.19	378.22	415.40	453.91
52	244.12	277.76	312.23	347.58	383.96	421.48	460.33
53	248.31	282.32	317.12	352.80	389.46	427.29	466.47
54	252.46	286.80	321.90	357.85	394.78	432.86	472.31
55	256.52	291.16	326.53	362.71	399.85	438.13	477.82
56	260.50	295.40	330.98	367.35	404.64	443.10	483.01
57	264.37	299.49	335.24	371.74	409.17	447.77	487.86
58	268.06	303.37	339.24	375.85	413.39	452.09	492.31
59	271.61	307.04	343.02	379.71	417.30	456.07	496.38
60	274.97	310.52	346.59	383.32	420.94	459.73	500.09



*Terminal Net Values per \$1,000 of Whole Life Policies by Twenty Equal Annual Premiums, Combined Experience, Four per Cent.*

AGE.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.	20th Year.
20	254.21	277.43	301.66	326.99	353.41	381.04
21	260.13	283.89	308.70	334.60	361.66	389.96
22	266.25	290.57	315.96	342.48	370.20	399.18
23	272.55	297.46	323.45	350.63	379.03	408.71
24	279.07	304.58	331.21	359.06	388.14	418.52
25	285.79	311.92	339.23	367.75	397.51	428.57
26	292.75	319.54	347.50	376.69	407.13	438.86
27	299.95	327.40	356.03	385.86	416.96	449.35
28	307.40	335.48	364.75	395.23	426.97	460.02
29	315.05	343.77	373.67	404.76	437.15	470.88
30	322.89	352.23	382.72	414.44	447.48	481.91
31	330.90	360.82	391.92	424.27	457.98	493.11
32	339.01	369.52	401.24	434.23	468.61	504.46
33	347.24	378.34	410.68	444.32	479.39	515.95
34	355.56	387.27	420.22	454.52	490.27	527.57
35	363.98	396.29	429.87	464.83	501.25	539.31
36	372.44	405.35	439.56	475.18	512.34	551.16
37	380.95	414.45	449.29	485.60	523.49	563.10
38	389.48	423.60	459.09	496.08	534.70	575.14
39	398.02	432.75	468.89	506.57	545.98	587.26
40	406.54	441.88	478.68	517.09	557.28	599.43
41	415.03	450.99	488.47	527.61	568.60	611.63
42	423.43	460.05	498.22	538.10	579.89	623.83
43	431.76	469.03	507.89	548.51	591.13	636.00
44	440.03	477.94	517.49	558.85	602.29	648.12
45	448.21	486.73	526.94	569.04	613.35	660.17
46	456.25	495.37	536.24	579.10	624.27	672.12
47	464.19	503.89	545.41	589.01	635.04	683.96
48	471.92	512.21	554.36	598.70	645.64	695.65
49	479.50	520.31	563.09	608.18	656.02	707.19
50	486.84	528.17	571.56	617.39	666.17	718.57
51	493.93	535.76	579.74	626.32	676.09	729.76
52	500.76	543.06	587.62	634.98	685.75	740.77
53	507.25	549.99	595.16	643.30	695.10	751.57
54	513.42	556.61	602.35	651.26	704.18	762.15
55	519.25	562.85	609.14	658.86	712.91	772.51
56	524.72	568.70	615.56	666.08	721.31	782.65
57	529.81	574.14	621.54	672.89	729.35	792.54
58	534.46	579.14	627.06	679.25	737.01	802.20
59	538.70	583.66	632.08	685.12	744.27	811.59
60	542.53	587.74	636.68	690.55	751.11	820.74



*Terminal Net Values per \$1,000 of Endowment Policies by Equal Annual Premiums Till Maturity, Payable at End of Twenty Years or at Death, if Prior, Combined Experience, Four per Cent.*

AGE.	1st Year.	2d Year.	3d Year.	4th Year.	5th Year.	6th Year.	7th Year.
20	31.39	64.19	98.49	134.33	171.80	210.99	251.99
21	31.38	64.18	98.45	134.28	171.74	210.92	251.89
22	31.38	64.16	98.42	134.24	171.69	210.84	251.80
23	31.37	64.14	98.40	134.20	171.63	210.76	251.70
24	31.36	64.13	98.36	134.15	171.56	210.68	251.58
25	31.35	64.11	98.33	134.10	171.49	210.58	251.46
26	31.34	64.09	98.29	134.04	171.41	210.47	251.32
27	31.34	64.06	98.25	133.98	171.32	210.35	251.18
28	31.32	64.04	98.21	133.91	171.23	210.24	251.05
29	31.32	64.02	98.17	133.86	171.15	210.15	250.93
30	31.31	64.00	98.14	133.81	171.09	210.06	250.82
31	31.31	63.98	98.11	133.77	171.04	210.00	250.76
32	31.30	63.97	98.10	133.76	171.02	209.98	250.73
33	31.31	64.00	98.13	133.79	171.06	210.03	250.79
34	31.33	64.03	98.18	133.86	171.14	210.13	250.91
35	31.36	64.08	98.26	133.97	171.29	210.31	251.12
36	31.40	64.18	98.40	134.15	171.52	210.59	251.45
37	31.46	64.30	98.59	134.41	171.85	210.97	251.84
38	31.54	64.46	98.84	134.75	172.26	211.42	252.27
39	31.65	64.69	99.17	135.18	172.74	211.89	252.70
40	31.79	64.97	99.58	135.66	173.23	212.37	253.10
41	31.96	65.28	99.99	136.12	173.70	212.77	253.41
42	32.14	65.59	100.38	136.53	174.08	213.08	253.65
43	32.31	65.86	100.71	136.85	174.36	213.31	253.80
44	32.44	66.10	100.96	137.09	174.57	213.47	253.87
45	32.57	66.27	101.16	137.29	174.74	213.58	253.91
46	32.66	66.39	101.31	137.44	174.87	213.65	253.89
47	32.75	66.58	101.54	137.68	175.06	213.78	253.91
48	32.85	66.76	101.76	137.90	175.26	213.91	253.95
49	32.98	66.97	102.01	138.16	175.49	214.08	254.03
50	33.11	67.19	102.29	138.45	175.76	214.29	254.13
51	33.25	67.44	102.59	138.78	176.08	214.55	254.32
52	33.41	67.71	102.95	139.18	176.46	214.90	254.59
53	33.58	68.04	103.36	139.64	176.95	215.36	254.96
54	33.80	68.40	103.84	140.20	177.54	215.92	255.40
55	34.02	68.81	104.41	140.87	178.24	216.56	255.91
56	34.28	69.29	105.07	141.64	179.02	217.26	256.46
57	34.60	69.88	105.84	142.49	179.87	218.03	257.07
58	34.95	70.50	106.63	143.38	180.76	218.84	257.70
59	35.33	71.14	107.47	144.30	181.68	219.67	258.35
60	35.69	71.80	108.30	145.24	182.63	220.52	259.00

*Terminal Net Values per \$1,000 of Endowment Policies by Equal Annual Premiums Till Maturity, Payable at End of Twenty Years or at Death, if Prior, Combined Experience, Four per Cent.*

AGE.	8th Year.	9th Year.	10th Year.	11th Year.	12th Year.	13th Year.
20	294.87	339.74	386.71	435.87	487.36	541.30
21	294.76	339.61	386.56	435.71	487.18	541.10
22	294.64	339.47	386.40	435.53	486.98	540.89
23	294.52	339.33	386.22	435.33	486.77	540.67
24	294.38	339.16	386.03	435.12	486.54	540.43
25	294.23	338.98	385.83	434.90	486.30	540.18
26	294.06	338.80	385.62	434.67	486.06	539.93
27	293.90	338.60	385.41	434.44	485.82	539.68
28	293.74	338.42	385.21	434.22	485.59	539.45
29	293.60	338.26	385.03	434.04	485.40	539.26
30	293.48	338.13	384.89	433.89	485.25	539.11
31	293.41	338.05	384.81	433.81	485.17	539.01
32	293.38	338.03	384.80	433.80	485.14	538.92
33	293.45	338.11	384.88	433.85	485.13	538.85
34	293.59	338.26	385.01	433.94	485.15	538.77
35	293.83	338.49	385.19	434.04	485.15	538.67
36	294.15	338.77	385.40	434.14	485.13	538.54
37	294.51	339.06	385.58	434.21	485.08	538.38
38	294.88	339.33	385.75	434.24	484.99	538.16
39	295.22	339.56	385.85	434.23	484.84	537.89
40	295.52	339.75	385.92	434.15	484.63	537.54
41	295.74	339.86	385.89	433.99	484.33	537.09
42	295.88	339.88	385.78	433.73	483.91	536.54
43	295.93	339.80	385.56	433.36	483.39	535.87
44	295.89	339.64	385.25	432.90	482.77	535.09
45	295.81	339.42	384.88	432.37	482.07	534.25
46	295.67	339.14	384.45	431.78	481.32	533.34
47	295.58	338.91	384.04	431.19	480.56	532.40
48	295.49	338.66	383.65	430.61	479.79	531.41
49	295.43	338.46	383.27	430.04	478.98	530.36
50	295.44	338.30	382.93	429.46	478.14	529.25
51	295.49	338.21	382.60	428.87	477.25	528.08
52	295.64	338.14	382.28	428.25	476.33	526.82
53	295.83	338.12	381.97	427.63	475.35	525.48
54	296.09	338.12	381.67	426.98	474.31	524.05
55	296.38	338.14	381.36	426.28	473.20	522.50
56	296.71	338.18	381.03	425.54	472.00	520.84
57	297.08	338.23	380.70	424.75	470.72	519.08
58	297.45	338.26	380.30	423.88	469.36	517.20
59	297.83	338.26	379.88	422.98	467.93	515.22
60	298.18	338.24	379.44	422.04	466.43	513.13

*Terminal Net Values per \$1,000 of Endowment Policies by Equal Annual Premiums Till Maturity, Payable at End of Twenty Years or at Death, if Prior, Combined Experience, Four per Cent.*

AGE.	14th Year.	15th Year.	16th Year.	17th Year.	18th Year.	19th Year.
20	597.81	657.06	719.18	784.35	852.75	924.57
21	597.61	656.86	718.99	784.19	852.63	924.49
22	597.40	656.65	718.80	784.02	852.49	924.41
23	597.17	656.42	718.58	783.83	852.35	924.33
24	596.93	656.18	718.36	783.64	852.20	924.25
25	596.67	655.93	718.13	783.44	852.05	924.15
26	596.42	655.69	717.90	783.24	851.89	924.06
27	596.17	655.45	717.68	783.04	851.72	923.95
28	595.95	655.24	717.47	782.84	851.55	923.84
29	595.76	655.04	717.26	782.63	851.36	923.72
30	595.59	654.85	717.05	782.41	851.16	923.59
31	595.45	654.66	716.83	782.17	850.95	923.45
32	595.31	654.46	716.58	781.91	850.71	923.29
33	595.16	654.24	716.32	781.63	850.46	923.13
34	594.99	654.00	716.02	781.32	850.18	922.94
35	594.79	653.72	715.69	780.97	849.87	922.74
36	594.56	653.41	715.32	780.58	849.52	922.51
37	594.29	653.05	714.90	780.15	849.13	922.26
38	593.97	652.63	714.42	779.66	848.70	921.98
39	593.58	652.14	713.87	779.10	848.22	921.67
40	593.10	651.57	713.24	778.47	847.67	921.32
41	592.53	650.89	712.51	777.75	847.06	920.93
42	591.84	650.11	711.68	776.95	846.37	920.50
43	591.04	649.22	710.75	776.04	845.60	920.01
44	590.14	648.22	709.71	775.04	844.74	919.46
45	589.16	647.15	708.58	773.93	843.79	918.86
46	588.11	645.97	707.34	772.72	842.75	918.20
47	586.99	644.71	706.00	771.41	841.63	917.48
48	585.80	643.35	704.55	769.99	840.39	916.69
49	584.53	641.90	702.99	768.44	839.06	915.83
50	583.17	640.33	701.30	766.77	837.60	914.88
51	581.71	638.64	699.47	764.94	836.00	913.86
52	580.15	636.82	697.48	762.95	834.27	912.73
53	578.47	634.84	695.32	760.80	832.38	911.50
54	576.65	632.71	693.00	758.47	830.33	910.16
55	574.69	630.42	690.49	755.94	828.10	908.70
56	572.60	627.96	687.79	753.21	825.68	907.10
57	570.38	625.33	684.89	750.27	823.05	905.36
58	568.00	622.51	681.77	747.08	820.19	903.45
59	565.47	619.51	678.42	743.64	817.10	901.36
60	562.80	616.32	674.85	739.96	813.75	899.09



## Valuation Columns.

Comb. Exp. 4 %.

$$u_x = \frac{D_x}{D_{x+1}}$$

$$k_x = \frac{C_x}{D_{x+1}}$$

AGE.	$u_x$	$k_x$	AGE.	$u_x$	$k_x$
20	1.047 638	0.007 344	60	1.072 537	0.031 285
21	1.047 729	0.007 432	61	1.075 060	0.033 711
22	1.047 821	0.007 520	62	1.077 855	0.036 399
23	1.047 927	0.007 622	63	1.080 901	0.039 328
24	1.048 034	0.007 725	64	1.084 266	0.042 563
25	1.048 144	0.007 831	65	1.087 959	0.046 115
26	1.048 267	0.007 949	66	1.091 994	0.049 994
27	1.048 393	0.008 071	67	1.096 438	0.054 268
28	1.048 534	0.008 206	68	1.101 263	0.058 907
29	1.048 678	0.008 344	69	1.106 486	0.063 928
30	1.048 836	0.008 497	70	1.112 220	0.069 442
31	1.048 999	0.008 653	71	1.118 470	0.075 452
32	1.049 177	0.008 824	72	1.125 303	0.082 022
33	1.049 359	0.008 999	73	1.132 754	0.089 186
34	1.049 546	0.009 179	74	1.140 936	0.097 054
35	1.049 750	0.009 375	75	1.149 883	0.105 657
36	1.049 959	0.009 576	76	1.159 652	0.115 050
37	1.050 173	0.009 782	77	1.170 472	0.125 453
38	1.050 406	0.010 005	78	1.182 415	0.136 938
39	1.050 644	0.010 235	79	1.195 491	0.149 511
40	1.050 889	0.010 471	80	1.209 874	0.163 340
41	1.051 155	0.010 726	81	1.225 599	0.178 461
42	1.051 455	0.011 014	82	1.242 821	0.195 020
43	1.051 834	0.011 379	83	1.262 002	0.213 463
44	1.052 309	0.011 836	84	1.283 441	0.234 078
45	1.052 858	0.012 363	85	1.308 333	0.258 012
46	1.053 526	0.013 006	86	1.337 587	0.286 141
47	1.054 249	0.013 701	87	1.372 456	0.319 669
48	1.055 045	0.014 466	88	1.415 494	0.361 052
49	1.055 903	0.015 291	89	1.469 720	0.413 192
50	1.056 845	0.016 197	90	1.537 848	0.478 700
51	1.057 876	0.017 189	91	1.627 509	0.564 912
52	1.059 006	0.018 275	92	1.748 673	0.681 416
53	1.060 243	0.019 464	93	1.916 087	0.842 391
54	1.061 564	0.020 735	94	2.150 112	1.067 416
55	1.063 030	0.022 144	95	2.501 622	1.405 405
56	1.064 621	0.023 674	96	2.960 000	1.846 154
57	1.066 316	0.025 304	97	3.380 000	2.250 000
58	1.068 185	0.027 101	98	4.160 000	3.000 000
59	1.070 230	0.029 068			













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